Zbl 015.24603
Erdős, Paul
On a problem of Chowla and some related problems. (In English)
Proc. Camb. Philos. Soc. 32, 530-540 (1936).

The problem in question is whether the integers n for which $d(n+1) > d(n)$ have density $\frac{1}{2}$, where $d(n)$ denotes the number of divisors of n. The author proves that this is the case. He first proves a general theorem to the effect that if $f(n)$ satisfies (1)$f(n) \geq 0$, (2) $f(mn) = f(m) + f(n)$ provided that $(m,n) = 1$, (3) $\sum f(p)/p$ (summed over all primes p) converges, then the density of the integers n for which $f(n+1) < f(n)$ is $\frac{1}{2}$. The method of proof is based on that used by the author in a previous paper (see Zbl 012.01004). The fundamental idea is that of approximating to $f(n)$ by

$$f_k(n) = \sum_{p<p_k} f(p^\alpha), \text{ where } p^\alpha \mid m, p^{\alpha+1} \nmid m.$$

The author then establishes that the result of the theorem holds also for $V(n)$, the number of different prime factors of n, which satisfies (1) and (2) but not (3). The proof of this is on the same lines but much more complicated, as k is taken to be a function of n of the order of magnitude $n^{(\log\log n)^{-3}}$. Finally the result for $d(n)$ is obtained from that for $V(n)$ by proving that for almost all n,

$$(d(n+1) - d(n))(V(n+1) - V(n)) > 0.$$

Davenport

Classification:
11N25 Distribution of integers with specified multiplicative constraints