Zbl 016.10604

Erdős, Pál; Turán, Pál

On interpolation. I. Quadrature- and mean-convergence in the Lagrange- interpolation. (In English)

Ann. of Math., II. Ser. 38, 142-155 (1937).

Let $\{\xi_n\}$ be a sequence on n points from $[-1,+1]$ varying with n; let $L_n(x)$ denote the sequence of Lagrange polynomials coinciding with a given R integrable function $f(x)$ at the points ξ_n. The authors are interested in the mean convergence

$$(*) \qquad \lim_{n\to\infty}\int_{-1}^{+1} |f(x)-L_n(x)|^p\,dx = 0$$

for $p=2$ and $p=1$. Let ξ_n be the zeros of the orthogonal polynomial $p_n(x)$ of degree n corresponding to the weight function $w(x)\geq\mu>0$. Then (*) holds with $p=2$. The same is true if we choose for ξ_n the zeros of $p_n(x)+A_np_{n-1}(x)+B_np_{n-2}(x)$, where A_n arbitrary real, $B_n\leq 0$. If $\int_{-1}^{+1} w(x)dx$ and $\int_{-1}^{+1} w(x)^{-1}\,dx$ exist and ξ_n is defined by the zeros of the linear combination mentioned, (*) holds with $p=1$. Finally the existence of a continous function $f(x)$ is proved for which (*) with $p=2$ does not hold provided that $\sum_{k=1}^n\int_{-1}^{+1} l_k(x)^2\,dx$ is unbounded; here $l_k(x)$ are the fundamental polynomials of the Lagrange interpolation corresponding to the set ξ_n.

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Classification:
41A05 Interpolation
42A15 Trigonometric interpolation