Zbl 022.00903
Erdős, Paul; Wintner, Aurel
Additive arithmetical functions and statistical independence. (In English)
Amer. J. Math. 61, 713-721 (1939).

Important results are obtained concerning additive functions, i.e. functions $f(n)$ which satisfy $f(n_1n_2)=f(n_1)+f(n_2)$, whenever $(n_1,n_2)=1$; so that $f(n)$ is determined by the values of $f(p^k)$, for all primes p and all k. It is shown that such a function has an asymptotic distribution function σ if and only if $\sum p^{-1}g(p)$ and $\sum' p^{-1}g(p)^2$ are convergent, when $g(p)=f(p)$ or $g(p)=1$ according as $|f(p)|<1$ or $|f(p)|\geq 1$. Furthermore, if σ_p is the asymptotic distribution function of the function $f_p(n)$, which is defined by $f_p(n)=f(p^k)$ if $p^k|n$ and $p^{k+1}\nmid n$, then σ is the infinite convolution of the σ_p and the above condition for the existence of σ is identical with the condition that this infinite convolution be convergent. The complete proof of which large parts are given in earlier publications [cf. *P. Erdős*, J. London Math. Soc. 13, 119-127 (1938; Zbl 018.29301)] is long and involves delicate operations with prime numbers related to Brunn's method.

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Classification:
11N60 Distribution functions (additive and positive multipl. functions)
11K65 Arithmetic functions (probabilistic number theory)