Zbl 083.03702
Erdős, Paul
On an elementary problem in number theory. (In English)
Can. Math. Bull. 1, 5-8 (1958).

Given $0 < x \leq y$, the author seeks an estimate of the smallest $f(x)$ so that there exist integers u, v satisfying (1) $0 \leq u,\ v < f(x)$ and $(x+u, y+v) = 1$. He proves that for every $\varepsilon > 0$ there exists arbitrarily large x satisfying

$$f(x) > (1-\varepsilon)(\log x/\log\log x)^{1/2}, \tag{2}$$

but for some $c > 0$ and all x, (3) $f(x) < c\log x/\log\log x$. The author indicates that it seems a difficult problem to get a sharp estimate of $f(x)$. He proves also the Theorem. Let $g(x)(\log x/\log\log x)^{-1} \to \infty,\ 0 \leq x < y$. Then the number of pairs $0 \leq u,\ v < g(x)$ satisfying $(x+u, y+v) = 1$ equals $(1+o(1))6\pi^{-2}g^2(x)$.

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Classification:
11N56 Rate of growth of arithmetic functions

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