Zbl 147.30201
Erdős, Pál
Some remarks on number theory. II (In English)
Isr. J. Math. 5, 57-64 (1967). [0021-2172]

[Part I cf. Zbl 131.03902]
According to the author's summary, this paper contains several disconnected remarks on number theory. The main results are:
Theorem 1. Let $f(k)$ be a real-valued arithmetical function, with

$$\lim_{n\to\infty} n^{-1}\sum_{k=1}^{n} f(k) = \alpha(\neq \pm\infty).$$

Assume that for every $\eta > 0$, there is a $g(\eta)$ so that for $l > g(\eta)$ and $n > 0$, $l^{-1}\sum_{k=0}^{l-1} f(n+k) < \alpha + \eta$; then to every $\varepsilon > 0$, $\delta > 0$, there is an $h(\varepsilon,\delta)$ so that for all but εx integers $n < x$, we have for every $l > h(\varepsilon,\delta)$ that

$$\left| l^{-1}\sum_{k=0}^{l-1} f(n+k) - \alpha \right| < \delta.$$

This generalizes (a strengthened form of) a result of *R.Bellman* and *H.N.Shapiro* (Zbl 057.28602).
Theorem 2. To every c_1, there is a $c_2(c_1)$, so that if $a_1 < a_2 < \cdots < a_k \leq n$ are integers, $k > c_1 n$, $A = a_1 a_2 ... a_n$, then $\sum_{d|A} d^{-1} > c_2 \log n$. The proof uses Brun's method.
Also the following result (not stated as a formal theorem) is proved: Let $a_1 < a_2 < \cdots < a_k \leq x$ be k integers such that no two of them are relatively prime, but every three are. If, for given x, one sets $\max k = f(x)$, then $f(x) = (\frac{1}{2} + o(1))(\log x)/(\log\log x)$.

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Classification:
11N64 Characterization of arithmetic functions