
Zbl 174.08104
Erdős, Pál
On the distribution of prime divisors (In English)
Aequationes Math. 2, 177-183 (1969). [0001-9054]

Denote by $v(n;a,b)$ the number of distinct prime factors of n satisfying $a \leq p \leq b$. $v(n) = v(n;l,n)$ denotes the number of distinct prime factors of n. A well known theorem of Hardy and Ramanujan states that for almost all $n, v(n) = (1+o(1)) \log\log n$. The principal aim of this paper is to prove that if $b - a/\log\log n \to \infty$ then for almost all integers n

$$v(n;a,b) = (1+o(1))(\log\log b - \log\log a)$$

uniformly in a and b. More precisely to every $\varepsilon > 0$ there is a c so that if we neglect $o(x)$ integers $n < x$ then for every a, b satisfying

$$\log\log b - \log\log a > c \log\log\log n \tag{1}$$

we have

$$(1-\varepsilon)(\log\log b - \log\log a) < v(n;a,b) < (1+\varepsilon)(\log\log b - \log\log a).$$

We also show that (1) is essentially best possible. The proof uses Turán's method and other ideas of probabilistic number theory. Some related results in probability and number theory are also discussed.

Classification:
11N25 Distribution of integers with specified multiplicative constraints
11N30 Turan theory