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On some extremal problems on r-graphs (In English)
Discrete Math. 1, 1-6 (1971). [0012-365X]

Denote by $G^{(r)}(n;k)$ an r-graph of n vertices and k r- tuples. Turán's classical problem states: Determine the smallest integer $f(n;r,l)$ so that every $G^{(r)}(n;f(n;r,l))$ contains a $K^{(r)}(l)$. Turán determined $f(n;r,l)$ for $r=2$, but nothing is known for $r>2$. Put $\lim_{n\to\infty} f(n;r,l)/\binom{n}{r} = c_{r,l}$. The values of $c_{r,l}$ are not known for $r>2$. I prove that to every $\epsilon>0$ and integer t there is an $n_0=n_0(t,\epsilon)$ so that every $G^{(r)}(n;[(c_{r,l}+\epsilon)(\binom{n}{r})])$ has lt vertices $x_i^{(j)}$, $1\le i\le t$, $1\le j\le l$, so that all the r-tuples $\left\{X_{i_1}^{(j_1)},\dots,X_{i_r}^{(j_r)}\right\}$, $1\le i_s\le t$, $1\le j_1<\dots<j_r\le l$, occur in our $G^{(r)}$. Several unsolved problems are posed.

Classification:
05C35 Extremal problems (graph theory)