Zbl 272.10003
Erdős, Paul
Über die Zahlen der Form $\sigma(n) - n$ und $n - \varphi(n)$.
On the numbers of the form $\sigma(n) - n$ und $n - \varphi(n)$. (In German)
Elemente Math. 28, 83-86 (1973).

In 1955, Sierpiński and the author discussed Euler's φ- function and believed that the equation (1) $n - \varphi(n) = m$ is insoluble for an infinity of values of m. Although no proof of this belief is yet available, the author considers the analogous problem: (2) $\sigma(n) - n = m$ is insoluble for an infinity of values of m. He proves the statement with the aid of two theorems. Theorem 1: The lower density of the numbers m for which (2) is insoluble is positive. Theorem 2: Given $\epsilon > 0$ there exists a k such that for all $x > x_0(\epsilon, k)$ the function $A(k, x)$ of numbers $n \neq$ prime p for which $\sigma(n) - n \leq x$, $\sigma(n) - n \equiv 0(\mod P_k)$ is valid is smaller than $\epsilon x/P_k$ where P_k denotes the product of the first k primes. He next proves a well-known Lemma: Let p be an arbitrary prime. The density of the numbers n with $\sigma(n) \not\equiv 0(\mod p)$ is 0.

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Classification:
11A25 Arithmetic functions, etc.
11B83 Special sequences of integers and polynomials

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