Zbl 272.41007
Erdős, Paul; Reddy, A.R.
Rational approximation to certain entire functions in $[0, +\infty)$. (In English)
Bull. Am. Math. Soc. 79, 992-993 (1973).

Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_0 > 0$ and $a_k \geq 0$ $(k \geq 1)$ be an entire function. Denote

$$\lambda_{0,n} = \lambda_{0,n}(1/f) = \inf_{p_n \in \pi_n} \left| \frac{1}{f(x)} - \frac{1}{p_n(x)} \right|_{L_\infty[0,\infty)}$$

where π_n denotes the clan of all polynomials of degree at most n. By using the above notation the authors announce the following. (i) For each $\epsilon > 0$, there exists infinitely many m for which $\lambda_{0,m} \leq \exp\left(\frac{-m}{(\log m)^{1+\epsilon}}\right)$. ii) For each $\epsilon > 0$ there exist a subsequence of natural numbers for which $\lambda_{0,m} \geq \exp(-\epsilon m)$. (iii) Let $a_j = q^{j^k}$, where $0 < q < 1$, $2 \leq k < \infty$. Then

$$q = \underline{\lim}(\lambda_{0,n})^{n^{1/k}} \leq \overline{\lim}(\lambda_{0,n})^{n^{k/1}} \leq q^{1-2^{1-k}}.$$

Classification:
41A20 Approximation by rational functions
41A50 Best approximation
41A25 Degree of approximation, etc.