
Zbl 339.10006

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On the number of distinct prime divisors of $\binom{n}{k}$. (In English)

Utilitas Math. 10, 51-60 (1976).

For positive integers n, k; $V(n,k)$ denotes the number of distinct prime divisors of $\binom{n}{k}$; m_k is the least n for which $V(n,k) = k$; n_k is the least n for which $V(n,k) \geq k$; and N_k is the least integer such that for every $n \geq N_k$, $V(n,k) \geq k$. It is likely (but not certain) that m_k exists for each k. The authors prove: (i) for $n > 4939$, $n_k > k^2$; (ii) for $n > 4949$, $n_k > k^2 \ln k$; (iii) for $k > k_0(c)$, $n_k > ck^2 \ln k$; (iv) $\liminf_{k\to\infty}(\ln n_k/\ln k) \leq e$; (v) for every $\epsilon > 0$ and $k > k_0(\epsilon)$, $N_k < (e+\epsilon)^k$. In Math. Scandinav. 39, 271- 281 (1976; Zbl 344.10003) and *Ernst S.Selmer* has given n_k for each $k \leq 200$. From his results, it appears that the only values for which $n_k \leq k^k$ are $k = 2, 3, \ldots, 30, 32, 36, 37$. The tables in the paper under review, record some interesting facts about $\binom{n}{k}$, e.g. $\binom{23}{k}$ is a product of distinct primes for each $k \leq 22$. Moreover m_k is not always $< m_{k+1}$ nor is n_k necessarily $\geq m_k$. Besides Selmer's paper, the reader might also refer to the reviewer's paper [Publ. Fac. Électrotechn. Univ. Belgrade, Sér. Mat. Phys. 498-541, 77-83 (1975; Zbl 315.10004)].

Classification:
11A41 Elemementary prime number theory
11B39 Special numbers, etc.
05A10 Combinatorial functions