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Prime polynomial sequences. (In English)

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Let $F(x)$ be a polynomial of degree $d \geq 2$ with integral coefficients and such that $F(n) \geq 1$ for all $n \geq 1$, Let $\mathfrak{G}_F = \{F(n)\}_{n=1}^{\infty}$. Then $F(n)$ is called composite in $\mathfrak{G}_F$ if $F(n)$ is the product of strictly smaller terms of $\mathfrak{G}_F$. Otherwise $F(n)$ is prime in $\mathfrak{G}_F$. It is proved that, if $F(x)$ is not of the form $a(bx+c)^d$, then almost all members of $\mathfrak{G}_F$ are prime in $\mathfrak{G}_F$. More precisely, if $C(x)$ denotes the number of composite $F(n)$ in $\mathfrak{G}_F$, with $n \geq x$, then, for any $\epsilon > 0$, it is shown that $C(x) \ll x^{1-(1/d^2)+\epsilon}$. For monic quadratics an identity implies that $C(x) \gg x^{\frac{1}{2}}$ so that in this case $x\frac{1}{2} \ll C(x) \ll x^{\frac{3}{4}+\epsilon}$. On the other hand, it is easy to construct polynomials for which $C(x) = 0$ for all x. In general, the exact order of $C(x)$ is unknown.

Classification:
11N13 Primes in progressions
11B83 Special sequences of integers and polynomials