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On a problem in extremal graph theory. (In English)
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From the authors introduction. Let $G(n,m)$ denote a graph (V,E) with n vertices and m edges and K_1 a complete graph with i vertices. *P. Turán* proved that every $G(n,T(n,k))$ contains a K_k, where

$$T(n,k)=\frac{k-2}{2(k-1)}(n^2-r^2)+\binom{r}{2}+1,$$

$r\equiv n(\mod k-1)$ and $0\le r\le k-2$.
The problem: For k a positive integer a graph $g(n,m)$ is said to posses property P(k) if $n\ge\binom{k+1}{2}$ and it contains vertex-disjoint subgraphs $K_1,K_2,\dots,K_k$. Find the least positive integer $T^*(n,k)$ such that every $G(n,T^*(n,k))$ has P(k). Clearly $T^*(n,k)\ge T(n,k)$. The following results are proved.
Theorem 1. If $n\ge 9k^2/k$ then $T^*(n,k)=T(n,k)$.
Theorem 2. There exists $\varepsilon>0$ and $k_0=k_0(\varepsilon)$ such that if $k>k_0$ and $\binom{k+1}{2}\le n\le\binom{k+1}{2}+\varepsilon k^2$ then $T^*(n,k)>T(n,k)$. Put $n=\binom{k+1}{2}+t$ and let $e(t,k)$ denote the number of edges of the n-vertex graph $X(t,k)$ whose complement consists of a K_{k+t+1} together with $n-k-t-1$ isolated vertices.
Theorem 3. There exists $k_0=k_0(t)$ such that if $k>k_0$ then $T^*(n,k)=e(t,k)+1$ and the only $G(n,e(t,k))$ without P(k) is $X(t,k)$.

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Classification:
05C35 Extremal problems (graph theory)