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On the asymptotic behavior of large prime factors of integers. (In English)

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The paper under review continues the work of Hardy and Ramanujan on round numbers and related topics. The authors study the functions $p_1(n)$, $p_2(n)\dots$ defined as the biggest prime factor of n, and the next smaller prime factor and so on. More precisely let $n=\prod_{i=1}^r p_i^{\alpha_i}$ where $p_1>p_2>\dots$ and $\alpha_i>0$. $A(n)=\Sigma\alpha_1$, $A^*(n)=\Sigma p_i$, $Omega(n)=\Sigma\alpha_1$, $\omega(n)=r$, $P_1(n)=P_1^*(n)=p_1$, $P_k^*(n)=p_k$ for $k\le\omega(n)$ and zero for $k>\omega(n)$, $P_k(n)=P_1\left(\frac{n}{P_1(n)P_2(n)\dots P_{k-1}(n)}\right)$ for $1<k\le\Omega(n)$ and zero for $k>\Omega(n)$. The functions $\Omega(n)$ and $\omega(n)$ were studied by Hardy and Ramanujan who proved that their normal order is $\log\log n$. The authors make a comparative study of A(n), $A^*(n)$, $P_k(n)$ and $P_k^*(n)$. With this in view they introduce

$$S_1(x,k)=\sum_{2\le n\le x}\frac{A(n)-P_1(n)-\dots-P_{k-1}(n)}{P_1(n)},$$

$$S_2(x,k)=\sum_{2\le n\le x}\frac{A^*(n)-P_1^*(n)-\dots-P_{k-1}^*(n)}{P_1(n)},$$

$$S_3(x,k)=\sum_{2\le n\le x}\frac{P_k(n)}{P_1(n)},\text{ and } S_4(k,x)=\sum_{2\le n\le x}\frac{P_k^*(n)}{P_1(n)},$$

where $k\ge1$ is any fixed positive integer. In anearlier paper they proved that as $x\to\infty$,

$$\sum_{1\le n\le x}(A(n)-P_1(n)-\dots-P_{k-1}(n))\sim\sum_{1\le n\le x}P_k(n)\sim$$

$$\sim\sum_{1\le n\le x}P_k^*(n)\sim\sum_{1\le n\le x}(A^*(n)-P_1^*(n)-\dots-P_{k-1}^*(n))\sim a_k(\log x)^{-k}x^{1+1/k},$$

where a_k is a positive constant depending on k. They also proved

$$\sum_{1\le n\le x}(A(n)-A^*(n))=x\log\log x+0(x).$$

To these they add another set of interesting results. Namely that as $x\to\infty$,

$$S_1(x,k)\sim S_2(x,k)\sim S_3(x,k)\sim S_4(x,k)\sim a_k'(\log x)^{1-k}x,$$

where a_k' is a positive constant depending only on k. These results give satisfactory information of the asymptotic behaviour (i.e. average order, normal order etc.) of the functions which they consider. For instance it is somewhat surprising that $A(n)$, $A^*(n)$, $P_1(n)$ are almost always nearly of the same order.

But none of them possess a normal order. The last mentioned result is deduced from the results mentioned above by appealing to a result of *P.D.T.A.Elliott*.

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Classification:
11N37 Asymptotic results on arithmetic functions
11N05 Distribution of primes

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Hardy-Ramanujan theorem; round numbers; large prime factors; average order; normal order