Let m and n be positive and let $f(n,m)$ denote the least integer so that in $(m, n + f(n,m)]$ there are distinct integers $a_1, \ldots, a_n$ such that $i|a_i$ for $i = 1, 2, \ldots n$. The authors study the function f(n,m) and in particular the function $f(n)$ defined by $f(n) = n + f(n,n)$. They establish some nice results on $f(n,m)$ and we state some of them. Theorem 2. For $n \geq 3$,

$$f(n) \geq (2e^{-\frac{1}{2}} + o(1))n \left(\frac{\log n}{\log \log n} \right)^{\frac{1}{2}}.$$

Theorem 3. Let r be the positive number satisfying $e^{-r} = r$. Then

$$f(n) \leq \left(\frac{r^{\frac{1}{2}}}{1 - r} + o(1) \right) n(\log n)^{\frac{1}{2}}.$$

Theorem 4. For all positive integers m, n we have $f(n,m) \leq 4n([n^{1/2}] + 1)$. Theorem 5. Let $\alpha = 1 - (\log(e \log 2))(\log 2)^{-1}$. Then

$$(L(n))^{-1} \sum_{m=1}^{L(n)} f(n,m) > n(\log n)^{\alpha},$$

$(n > n_0)$, where $L(n)$ denotes the l.c.m. of $1, 2, \ldots, n$ and n_0 is a certain numerical constant. Theorem 6. Let $\beta = \log(5^{5/2} 3^{-3/2} 2^{-1})$. Then

$$(L(n))^{-1} \sum_{m=1}^{L(n)} f(n,m) \leq n \exp((\beta + o(1))(\frac{\log n}{\log \log n})).$$

The proofs are fairly involved and a detailed study of the paper would be rewarding in the opinion of the reviewer. Some of the ideas involved in the proof are marriage theorem, some results of de Bruijn, some results of Tenenbaum.

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