Zbl 456.10024
Erdős, Paul; Katai, I.
On the maximal value of additive functions in short intervals and on some related questions. (In English)
Acta Math. Acad. Sci. Hung. 35, 257-278 (1980); correction ibid. 37, 499 (1981). [0001-5954]

Let $g(n)$ be a non-negative strongly additive function, $f_k(n) = \max_{j=i,\dots,k} g(n+j)$,

$$\rho(k,\varepsilon) = \sup_{x\geq 1} \frac{1}{x} \text{card}\{n \leq x | f_k(n) > (1+\varepsilon) f_k(0)\}$$

$$\delta(k_0,\varepsilon) = \sup_{x\geq 1} \frac{1}{x} \text{card}\{n \leq x | \exists k, k > k_0, f_k(n) > (1+\varepsilon) f_k(0)\}$$

and let

$$\theta(k,\varepsilon) = \lim\sup_{x=\infty} \frac{1}{x} \text{card}\{n \leq x | f_k(n) > (1+\varepsilon) f_k(0)\}.$$

One notes that $\theta(k,\varepsilon) \leq \rho(k,\varepsilon)$ and $\sup_{k\geq k_0} \rho(k,\varepsilon) \leq \delta(k_0,\varepsilon)$. It is shown that if $\rho(k,\varepsilon) \to 0 (k \to \infty)$ for all $\varepsilon > 0$, then $\sum_p \frac{g(p)^r}{p} < \infty$ for every $r \geq 1$, thus considerably stregthening a result previously obtained by the authors [Acta Math. Acad. Sci. Hung. 33, 345-359 (1979; Zbl 417.10039)]. Among other results obtained (space limitations unfortunately preclude giving the complete list), we find the following: If $g(p) = 1/p$ for promes p then

$$\sup_{x\geq 1} \frac{1}{x} \text{card}\{n \leq x | \exists k > k_0, f_k(n) > f_k(0) + \lambda_k\} \to 0, (k_0 \to \infty),$$

where $\lambda_k = 3/\log\log k$, while if $g(p) = 1(p^\delta$, $0 < \delta < 1$, and $\rho > 0$ is an arbitrary constant, then

$$\lim_{k\to\infty} \lim_{x\to\infty} \frac{1}{x} \text{card}\left\{n \leq x | f_k(n) > f_k(0) + (\log k)^{1-\delta-\rho}\right\} = 1.$$

In the paper reviewed above the authors stated erroneously that Theorem 1 is a consequence of Theorem 1'. In fact, the converse implication is true: Theorem 1 implies Theorem 1'. A proof of Theorem 1 is given.

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Classification:
11N37 Asymptotic results on arithmetic functions
11K65 Arithmetic functions (probabilistic number theory)

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distribution of maximal value; strongly additive function