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Erdős, Paul; Mills, George

Some bounds for the Ramsey-Paris-Harrington numbers. (In English)

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"It has recently been discovered that a certain variant of Ramsey's theorem cannot be proved in first-order Peano arithmetic although it is in fact a true theorem. in this paper er give some bounds for the "Ramsey-Paris- Harrington numbers" associated with the variant of Ramsey's theorem, involving coloring of pairs. In the course of the investigation we also study certain weaker and stronger partition relations." Let k be a fixed positive integer. For $n > k$, let $[k, n]$ denote $\{k, k+1, \dots, n\}$; for any set X let X^2 denote the collection of two element subsets of X. A two coloring of $[k, n]^2$, $F : [k, n]^2 \to \{1, 2\}$ is proper if there exists $Y \subseteq [k, n]$ and a color $1 \in \{1, 2\}$ such that: (i) $F(\{a, b\}) = i$ for all $\{a, b\} \in Y^2$; (ii) $|Y| \geq \min\{a|a \in Y\} \cup \{3\}$. The integer n is proper if all two coloring of $[k, n]^2$ are proper. $R(k)$ is then the minimum proper n. The authors compute: $R(1) = 6$, $R(2) = 8$, $R(3) = 13$, and $R(4) \leq 687$. They prove: (i) There exists $c > 0$ such that $(c\sqrt{k}/\log k)^{2^{k/2}} < R(k)$ for all sufficiently large k; (ii) $R(k) < 2^{k^{2k}}$ for all $k \geq 2$.

J.E.Graver

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