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Elekes, G.; Erdős, Paul; Hajnal, András
On some partition properties of families of sets. (In English)
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The paper states (without detailed proofs) results and problems concerning the existence of certain types of homogeneous sets for partitions $P(\kappa) = \cup\{D_\alpha; \alpha < \mu\}$ of the power set $P(\kappa)$ of the infinite cardinal κ into μ classes. We say $H \subseteq P(\kappa)$ is homogeneous for the partition if there is some $\alpha < \mu$ with $H \subseteq D_\alpha$. The first questions discussed concern homogeneous Δ- systems. The family $\dashv$ is called a λ, Δ-system if $|a| = \lambda$ and $A \cap B$ is the same for all distinct A, B from $\dashv$. Results stated include: For any partition of $P(\kappa)$ into κ classes and any cardinal $\delta < \kappa$, there is a homogeneous λ, Δ-system. If κ is regular this holds for $\lambda = \kappa$ as well. Further questions relate to homogeneous (λ, μ)-systems. The family $\mathcal{J}$ is said to bee a (λ, μ)-system if there is a family $\dashv$ with $|a| = \lambda$ such that $\mathcal{J}$ is the collection of all non- empty unions of $< \mu$-size subfamilies of $\dashv$ these unions being different for different subfamilies. Typical results: For any $\lambda < \kappa$ and any finite n, every partition of $P(\kappa)$ into κ classes has a homogeneous λ, n-system. If κ is regular, this holds for $\lambda = \kappa$ as well. If $2^{<\kappa} = \kappa$ any such partition has a homogeneous $\aleph_0, \aleph_0$-system, but $2^\kappa = \kappa^+$ then there is such a partition with no $\aleph_1, \aleph_0$-system.

N.H. Williams

Classification:
04A20 Combinatorial set theory

Keywords:
partitions of the power set of an infinite cardinal; homogeneous sets