Zbl 478.10027

Bateman, Paul T.; Erdős, Paul; Pomerance, Carl; Straus, E.G.

The arithmetic mean of the divisors of an integer. (In English)

Analytic number theory, Proc. Conf., Temple Univ./Phila. 1980, Lect. Notes Math. 899, 197-220 (1981).

[For the entire collection see Zbl 465.00008.]

This paper establishes the following interesting and deep results about the arithmetic function A, defined by $A(n) = \sigma(n)/d(n)$, i.e. $A(n)$ is the arithmetic mean of the divisors of n: If $N(x)$ denotes the number of integers n with $n \leq x$ and $A(n)$ not an integer, then

$$N(x) = x \exp\left(-(1+o(1))2\sqrt{\log 2}\sqrt{\log\log x}\right), \tag{1}$$

$$\sum_{n\leq x} A(n) \sim cx^2(\log x)^{-1/2}, \text{ with c an explicity given constant,} \tag{2}$$

$$\sum_{A(n)\leq x} 1 \sim \lambda x \log x, \text{ again with } \lambda \text{ an explicity given constant.} \tag{3}$$

Another teorem, in connection with (1), is the following: Denote for every positive real number β the number $\prod_{p^a||n} p^{[\alpha\beta]}$ by $\langle n^\beta\rangle$. Then for any ε between 0 and 2, the set of integers n for which $\langle d(n)^{2-\varepsilon}\rangle/sigma(n)$ has asymptotic density 1, the set of n for which $\langle d(n)^{2+\varepsilon}\rangle/\sigma(n)$ has asymptotic density 0, and the set of n for which $d(n)^2/\sigma(n)$ has asymptotic desity 1/2. The proofs are long and complicated, with applications of results from various parts of number theory. To mention only a few: sieve methods, the generalized Erdős-Kac theorem and Tauberian theorems of Delange.

H.Jager

Classification:
11N37 Asymptotic results on arithmetic functions
11N05 Distribution of primes

Keywords:
divisor function; sum of divisor function; arithmetic mean of divisors; asymptotic density