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Some asymptotic formulas on generalized divisor functions. IV. (In English)

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The paper under review is a sequel to three papers of the authors [Part I, Studies in pure Mathematics, Mem. of P. Turán, 165-179 (1983), II. J. Number Theory 15, 115-136 (1982; Zbl 488.10043) and III. Acta Arith. 41, 395- 411 (1982; Zbl 492.10037)]. For a given sequence A: $a_1 < a_2 < \dots$ of positive integers define $N_A(x) = \sum_{a\in A, a\le x} 1$, $f_A(x) = \sum_{a\in A, a\le x} \frac{1}{a}$, the divisor function $\tau_A(n) = \sum_{a\le A, a|n} 1$, and its maximum $D_A(x) = \max_{1\le n\le x} \tau_A(n)$. The authors are interested in "large" values of $D_A(x)$ compared with $f_A(x)$, the normal order of $\tau_A(n)$. For example, in III the authors showed that for all $\Omega > 0$, and for $x > X_0(\Omega)$ $D_A(x)/f_A(x) > \Omega$, if $f_A(X) > (\log\log x)^{20}$. Now it is shown that for any $\Omega > 1$ there exist constants $c_3(\Omega)$, $X_1(\Omega)$ such that $D_A(x)/f_A(x) > \Omega$, if $x > X_1$, if $f_A(x) > c_3$ and if the interval $[x^{1-\{f_A(x)\}^{-1/3}}, x]$ has void intersection with A. As the authors show, this theorem does not remain true, if $1 - \{f_A(x)\}^{-1/3}$ is replaced by $1 - c_6^{-f_A(x)}$.

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