
Zbl 563.10002
Erdős, Paul
Miscellaneous problems in number theory. (In English)
Numerical mathematics and computing, Proc. 11th Manitoba Conf., Winnipeg/Manit. 1981, Congr. Numerantium 34, 25-45 (1982).

[For the entire collection see Zbl 532.00008.]

Let $n! = \prod_{p_i} p_i^{\alpha_i(n)}$ be the prime factor decomposition of n! into distinct prime powers. *J.L.Selfridge* and the author proved the interesting Theorem. Denote by $h(n)$ the number of distinct exponents $\alpha_i(n)$. There are absolute positive constants c_1 and c_2 for which

$$c_1(n/\log n)^{1/2} < h(n) < c_2(n/\log n)^{1/2}.$$

The author conjectures that there exists a constant $c > 0$ such that $h(n) = (c+o(1))(n/\log n)^{1/2}$. Then he makes some conjectures about the prime factor decomposition of $\prod_{i=1}^{n}(x+i)$.
Next he proves the following Theorem. Let $(1+\epsilon)n < a_1 < a_2 < ... < a_k$, $(a_1...a_k)/n! = I_n$ where I_n has all its prime factors $\leq n$. Further let $a_k - a_1 < n$. Then $a_1 > 2^{n-c_3nL}$ where $L = \log\log n/\log n$. Finally some results on additive number theory are given.

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Classification:
11-02 Research monographs (number theory)
11A41 Elemementary prime number theory
11N37 Asymptotic results on arithmetic functions
11B13 Additive bases
11P99 Additive number theory
00A07 Problem books

Keywords:
disjoint sets of positive integers; distinct sum; unconventional problems; consecutive integers; factorial; prime factor decomposition; prime factors