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**Zbl 609.10034**

**Erdős, Paul; Pomerance, C.; Sárközy, A.**

*On locally repeated values of certain arithmetic functions. II.* (In English)

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[For part I, cf. J. Number Theory 21, 319-332 (1985; Zbl 574.10012).]
It seems reasonable, in view of the Erdős-Kac theorem, to conjecture that the number of $n \le x$ with $\nu(n) = \nu(n+1)$ is of exact order $x(\log\log x)^{-}$. The principal result of the paper is that there is a constant c such that $|\nu(n) - \nu(n+1)| \le c$ has at least order $x(\log\log x)^{-}$ solutions $n \le x$. It is stated that $c = 3$ is admissible. The proof uses the Erdős- Kac theorem to produce many pairs $a, b$ with $\nu(a) = \nu(b)$. One then takes $a_0, b_0$ to solve $ab_0 - ba_0 = 1$, and uses the small sieve to find integers $m$ for which $\nu(am + a_0)$, $\nu(bm + b_0)$ are both small. One may then take $n = b(am + a_0)$. The reviewer [Mathematika 31, 141-149 (1984; Zbl 529.10040)] showed by a different approach that the related equation $d(n) = d(n+1)$ (where $d(n)$ is the divisor function) has infinitely many solutions. It seems likely that one could handle $\nu(n) = \nu(n+1)$ the same way, but as yet there seems no hope of "hybridizing" the two methods so as to solve the original conjecture.
In the final section of the paper, the number of solutions $n \le x$ of the equation $\phi(n) = \phi(n+1)$, (where $\phi(n)$ is Euler's function) is considered, and shown to be at most $x/\exp((\log x)^{1/3})$. The proof is closely related to *C. Pomerance*'s treatment [J. Reine Angew. Math. 325, 183-188 (1981; Zbl 448.10007)] of amicable numbers.

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11N35 Sieves
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