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Erdős, Paul; Ivić, A.; Pomerance, C.
On sums involving reciprocals of the largest prime factor of an integer. (In English)
Glas. Mat., III. Ser. 21(41), 283-300 (1986). [0017-095X]

Let $P(n)$ denote the largest prime factor of n. Let $\rho(u)$ denote the continuous solution to the differential delay equation $u\rho'(u)=-\rho(u-1)$ with the initial condition $\rho(u)=1$ for $0\le u\le 1$. ($\rho(u)$ is usually called the Dickman-de Bruijn function). Let $\delta(x)=\int_2^x\rho(\log x/\log t)t^{-2}dt$. Then the authors prove

$$\sum_{n\le x}(P(n))^{-1}=x\delta(x)(1+O((\log\log x/\log x)^{1/2})).$$

The authors prove many other interesting results. We quote one or two results. Let $w(n)=\sum_{p|n}1$ and $\Omega(n)=\sum_{p^\alpha||n}$. Then the authors prove a rather surprising result

$$\sum_{n\le x}(P(n))^{-w(n)}=\exp\{(4+o(1))(\log x)^{1/2}(\log\log x)^{-1}\}$$

whereas (in contrast)

$$\sum_{n\le x}(P(n))^{-\Omega(n)}=\log\log x+D+O(1/\log x)$$

where D is a constant.
Another result which I would like to quote is about K. Alladi's functions $\beta(n)=\sum_{p|n}p$ and $\beta(n)=\sum_{p^\alpha||n}\alpha p$. If $f(n)$ denotes either of these functions then

$$\sum_{n\le x}1/f(n)=\{1+0(\exp(-C(\log x\log\log x)^{1/2}))\}\sum_{2\le n\le x}1/P(n).$$

The reader would find many other interesting results.

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Classification:
11N05 Distribution of primes
11N37 Asymptotic results on arithmetic functions

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reciprocals of largest prime factor; Dickman-de Bruijn function