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Extremal problems for degree sequences. (In English)

Combinatorics, Proc. 7th Hung. Colloq., Eger/Hung. 1987, Colloq. Math. Soc. János Bolyai 52, 183-193 (1988).

[For the entire collection see Zbl 673.00009.]

Let G be a graph of order n with degree sequence $(d_1\ led_2 \leq \ldots \leq d_n)$. Let $D = \{i : d_i = d_j$ for some $i \neq j\}$ (duplicated degrees) and $S = \{1, \ldots, n\} - D$ (single degrees). Let D' be that proper subset of D which one obtains in choosing the first index associated with each duplicated degree. Finally let $M = \{j : 0 \leq j \leq n-1$ and $j \neq d_i$ for any $i\}$ (missing degrees). It is proved the following location of duplicated degrees:

If $d_i \in S$ for all $i > k$, then $k \geq (\sqrt{4(n-\delta)+1}+1)/2$ where $\d{e}lta = 0$ for n even, and $\delta = 1$ for n odd, and this result is best possible. Let ΣD and $\Sigma D'$ be the sum of the degrees indexed by each of the sets. There are given bounds for these numbers, e.g.: If G has no isolated vertices, then $\Sigma D' \geq 1$ and $\Sigma D \geq \frac{n+3}{3}$ and these bounds are sharp. Furthermore let ΣM be the sum of the elements in M. For sufficiently large n one has $\Sigma D' + \Sigma M \geq \frac{n^{2/3}}{2}$ and $\Sigma D + \Sigma M \geq n/2 + n^{2/3}/2$ and these bounds are sharp in order of magnitude.

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