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Independent transversals in sparse partite hypergraphs. (In English)

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An $[n, k, r]$-hypergraph is any hypergraph with a kn-element set of vertices and whose edges can be obtained in the following manner: (1) partition the set of vertices into n k-element sets $V_1, \ldots, V_n$; (2) for any r-element subfamily $\{V_{i_1}, \ldots, V_{i_r}\}$ form an edge by picking from each V_{i_j} exactly one element. An independent transversal is a set of vertices which meets each V_i in exactly one point and does not contain any edge. The paper provides estimates for the function $f_r(k)$—the largest n for which any $[n, k, r]$-hypergraph has an independent transversal. In particular, in the case when $r = 2$, it is proved that $(1 + o(1))(2e)^{-1}k^2 < f_2(k) < (1 + o(1))k^2$. For those values of k for which an affine plane of order $k + 1$ exists, it is shown that $f_2(k) < (k + 1)^2$. Asymptotics for $f_r(k)$, in several cases when k is small compared to r, are also given.

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05D15 Transversal (matching) theory
05C65 Hypergraphs

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