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On the number of q-expansions. (In English)
Ann. Univ. Sci. Budap. Rolando Eötvös, Sect. Math. 37, 109-118 (1994). [0524-9007]

Let (p_i) be a sequence of positive numbers with $P = \sum p_i < \infty$. For a real number $x \in [0, P]$, let (c_i) and (d_i) be two sequences defined as follows: $c_1 = 1$ if $p_1 \leq x$, $c_1 = 0$ otherwise; if $c_1, \dots, c_{i-1}$ are already defined, let $c_i = 1$ if $c_1 p_1 + \dots + c_{i-1} p_{i-1} \leq x - p_i$, $c_i = 0$ otherwise; $d_1 = 0$ if $\sum_{j>1} p_j \geq x$, $d_1 = 1$ otherwise; if $d_1, \dots, d_{i-1}$ are already defined, let $d_i = 0$ if $\sum_{j>i} p_j \geq x - \sum_{j<i} p_j$, $d_i = 1$ otherwise.
If $\sum c_i p_i = x$ ($\sum d_i p_i = x$), then $\sum c_i p_i$ ($\sum d_i p_i$) is called the greedy (lazy) expansion of x. More generally, $\sum a_i p_i$ is an expansion of x if $a_i \in \{0, 1\}$ for every i and if $\sum a_i p_i = x$.
The authors investigate these expansions in case $p_i = q^{-i}$, where $q \in (1, 2)$ (q-expansions) and they give a new proof of the following property stated by the same authors [Bull. Soc. Math. Fr. 118, 377-390 (1990; Zbl 721.11005)]: For every $1 \leq N \leq \omega$ there are 2^ω numbers $q \in (1, 2)$ such that 1 has exactly N different q-expansions.

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expansions of real numbers; greedy expansion