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**Zbl 846.03023****Erdős, Paul; Hajnal, A.; Larson, Jean A.***Ordinal partition behavior of finite powers of cardinals.* (In English)**Sauer, N. W. (ed.) et al., Finite and infinite combinatorics in sets and logic. Proceedings of the NATO Advanced Study Institute, Banff, Canada, April 21-May 4, 1991. Dordrecht: Kluwer Academic Publishers, NATO ASI Ser., Ser. C, Math. Phys. Sci. 411, 97-115 (1993). [ISBN 0-7923-2422-6/hbk]**

In the notation of Erdős and Rado, the expression  $\alpha \rightarrow (\beta, p)^2$  means that for any graph on  $\alpha$  either there is an independent subset of type  $\beta$  or there is a complete subgraph of size  $p$ . We discuss results for this relation where  $\alpha$  and  $\beta$  are both finite powers of some cardinal. In particular, assume that  $\lambda$  is either a regular cardinal or a strong limit cardinal and that  $k$  and  $\ell$  are positive integers. Then  $\lambda^{1+k\ell} \rightarrow (\lambda^{1+k}, \ell+1)^2$ . On the other hand,  $\lambda^{k\ell} \not\rightarrow (\lambda^{1+k}, 2^{\ell-1} + 1)^2$  holds provided  $k \geq 4$ . We prove that the positive result is sharp if  $\lambda$  is a successor cardinal of the form  $\lambda = \theta^+ = 2^\theta$ , while the negative result is sharp if the cofinality of  $\lambda$  is a weakly compact cardinal.

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03E05 Combinatorial set theory (logic)

03E10 Ordinal and cardinal arithmetic

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finite powers of cardinals; partition ordinals; graph; independent subset; complete subgraph; regular cardinal; strong limit cardinal; successor cardinal