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Erdős, Paul; Saias, Eric

Sur le graphe divisoriel.

On the divisor graph. (In French)

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Let R_f, R_g be the relations on the positive integers not exceeding x given by

$$aR_fb \Leftrightarrow a|b \text{ or } b|a, \quad aR_gb \Leftrightarrow \operatorname{lcm}(a,b) \le x.$$

If $n_iR_fn_{i+1}$ for $i = 1, 2, \dots, \ell - 1$, then $n_1, n_2, \dots, n_\ell$ is said to be a chain of length ℓ for the relation R_f. Let $f(x)$ denote the maximum value of ℓ, and define similarly a quantity $g(x)$ for the relation R_g. The second author [Applications des entiers à diviseurs denses (Preprint)] showed that for all $x \ge 2$, $cx/\log x \le f(x) \le g(x) \le c'x \log x$ for certain positive constants c, c'. In this paper, the authors consider the minimum number $\phi(x)$ of chains for the relation R_f that are required in order that every positive integer $\le x$ belongs to at least one such chain, and the corresponding number $\gamma(x)$ for the relation R_g. The analogous quantity when the chains for R_f, R_g are pairwise disjoint is denoted by $\phi^*(x)$, $\gamma^*(x)$, respectively. It is shown that there exist positive constants c_1, c_2, c_3 such that for all $x \ge 2$,

$$\frac{c_1x}{\log x} \le \gamma(x) \le \phi(x) \le \frac{c_2x}{\log x}, \quad c_3x \le \gamma^*(x) \le \phi^*(x) \le \frac{x}{2}.$$

Although the proofs are elementary, the establishment of the upper bound for $\phi(x)$, particularly, is somewhat intricate.

E.J.Scourfield (Egham)

Classification:

11N56 Rate of growth of arithmetic functions

11N25 Distribution of integers with specified multiplicative constraints

11N37 Asymptotic results on arithmetic functions

Keywords:

divisor graph; relation in terms of least common multiple; chains for a relation; upper bound for the minimum number of chains

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