Uncle Paul[1]

Paul Erdős was a searcher, a searcher for mathematical truth.

Paul's place in the mathematical pantheon will be a matter of strong debate for in that rarefied atmosphere he had a unique style. The late Ernst Straus said it best, in a commemoration of Erdős's 70-th birthday.

> In our century, in which mathematics is so strongly dominated by "theory constructors" he has remained the prince of problem solvers and the absolute monarch of problem posers. One of my friends - a great mathematician in his own right - complained to me that "Erdős only gives us corollaries of the great metatheorems which remain unformulated in the back of his mind." I think there is much truth to that observation but I don't agree that it would have been either feasible or desirable for Erdős to stop producing corollaries and concentrate on the formulation of his metatheorems. In many ways Paul Erdős is the Euler of our times. Just as the "special" problems that Euler solved pointed the way to analytic and algebraic number theory, topology, combinatorics, function spaces, etc.; so the methods and results of Erdős's work already let us see the outline of great new disciplines, such as combinatorial and probabilistic number theory, combinatorial geometry, probabilistic and transfinite combinatorics and graph theory, as well as many more yet to arise from his ideas.

Straus, who worked as an assistant to Albert Einstein, noted that Einstein chose physics over mathematics because he feared that one would waste one's powers in persuing the many beautiful andattractive questions of mathematics without finding the central questions. Straus goes on,

> Erdős has consistently and successfully violated every one of Einstein's prescriptions. He has succumbed to the seduction of every beautiful problem he has encountered - and a great many have

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> succumbed to him. This just proves to me that in the search for truth there is room for Don Juans like Erdős and Sir Galahad's like Einstein.

I believe, and I'm certainly most prejudiced on this score, that Paul's legacy will be strongest in Discrete Math. Paul's interest in this area dates back to a marvelous paper with George Szekeres in 1935 but it was after World War II that it really flourished. The rise of the Discrete over the past half century has, I feel, two main causes. The first was The Computer, how wonderful that this physical object has led to such intriguing mathematical questions. The second, with due respect to the many others, was the constant attention of Paul Erdős with his famous admonition "Prove and Conjecture!" Ramsey Theory, Extremal Graph Theory, Random Graphs, how many turrets in our mathematical castle were built one brick at a time with Paul's theorems and, equally important, his frequent and always penetrating conjectures.

My own research specialty, The Probabilistic Method, could surely be called The Erdős Method. It was begun in 1947 with a 3 page paper in the Bulletin of the American Math Society. Paul proved the existence of a graph having certain Ramsey property without actually constructing it. In modern language he showed that an appropriately defined random graph would have the property with positive probability and hence there must exist a graph with the property. For the next twenty years Paul was a "voice in the wilderness", his colleagues admired his amazing results but adaption of the methodology was slow. But Paul persevered - he was always driven by his personal sense of mathematical aesthetics in which he had supreme confidence - and today the method is widely used in both Discrete Math and in Theoretical Computer Science.

There is no dispute over Paul's contribution to the spirit of mathematics. Paul Erdős was the most inspirational man I have every met. I began working with Paul in the late 1960-s, a tumultuous time when "do your own thing" was the admonition that resonated so powerfully. But while others spoke of it, this was Paul's modus operandi. He had no job; he worked constantly. He had no home; the world was his home. Possessions were a nuisance, money a bore. He lived on a web of trust, travelling ceaselessly from Center to Center, spreading his mathematical pollen.

What drew so many of us into his circle. What explains the joy we have in speaking of this gentle man. Why do we love to tell Erdős stories. I've thought a great deal about this and I think it comes down to a matter of

belief, or faith. We mathematicians know the beauties of of our subject and we hold a belief in its transcendent quality. God created the integers, the rest is the work of Man. Mathematical truth is immutable, it lies outside physical reality. When we show, for example, that two n-th powers never add to an n-th power for $n \geq 3$ we have discovered a Truth. This is our belief, this is our core motivating force. Yet our attempts to describe this belief to our nonmathematical friends are akin to describing the Almighty to an atheist. Paul embodied this belief in mathematical truth. His enormous talents and energies were given entirely to the Temple of Mathematics. He harbored no doubts about the importance, the absoluteness, of his quest. To see his faith was to be given faith. The religious world might better have understood Paul's special personal qualities. We knew him as Uncle Paul.

I do hope that one cornerstone of Paul's, if you will, theology will long survive. I refer to The Book. The Book consists of all the theorems of mathematics. For each theorem there is in The Book just one proof. It is the most aesthetic proof, the most insightful proof, what Paul called The Book Proof. And when one of Paul's myriad conjectures was resolved in an "ugly" way Paul would be very happy in congratulating the prover but would add, "Now, let's look for The Book Proof." This platonic ideal spoke strongly to those of us in his circle. The mathematics was there, we had only to discover it.

The intensity and the selflessness of the search for truth were described by the write Jorge Luis Borges in his story The Library of Babel. The narrator is a worker in this library which contains on its infinite shelves all wisdom. He wanders its infinite corridors in search of what Paul Erdős might have called The Book. He cries out,

> To me, it does not seem unlikely that on some shelf of the universe there lies a total book. I pray the unknown gods that some man - even if only one man, and though it have been thousands of years ago! - may have examined and read it. If honor and wisdom and happiness are not for me, let them be for others. May heaven exist though may place be in hell. Let me be outraged and annihilated but may Thy enormous Library be justified, for one instant, in one being.

In the summer of 1985 I drove Paul to what many of us fondly remember as Yellow Pig Camp - a mathematics camp for talented high school students at Hampshire College. It was a beautiful day - the students loved Uncle Paul

and Paul enjoyed nothing more than the company of eager young minds. In my introduction to his lecture I discussed The Book but I made the mistake of discribing it as being "held by God". Paul began his lecture with a gentle correction that I shall never forget. "You don't have to believe in God," he said, "but you should believe in The Book."

– Joel Spencer