Introduction

Erdős' love for geometry and elementary or discrete geometry in particular, dates back to his beginnings. The Erdős-Szekeres paper has been influential and certainly helped to create discrete geometry as we know it today. But Erdős also put geometry to the service to other branches, giving definition to various geometrical graphs and proving bounds on their chromatic and independence numbers. We are happy to include papers by Moshe Rosenfeld, Pavel Valtr, Janos Pach, Jiří Matoušek and, in particular, a paper by Miklós Laczkovich and Imre Ruzsa on the number of homothetic sets. While the paper of Peter Fishburn is closely related to Erdős' favorite theme, the papers of N. G. de Bruijn (on Penrose tiling) and J. Aczél and L. Losonczi (on functional equations) cover broader related aspects. It is perhaps fitting to complement this introduction by a few related Erdős problems in his own words:

Let $x_1, \ldots, x_n$ be n points in the plane, not all on a line, and join every two of them. Thus we get at least n distinct lines. This follows from Gallai-Sylvester but also from a theorem of de Bruijn and myself.

My most striking contribution to geometry is no doubt my problem on the number of distinct distances. This can be found in many of my papers on combinatorial and geometric problems.

Hickerson, Pach and I proved that on the unit sphere one can find n points for which the distance $\sqrt{2}$ can occur among $n^{1/3}$ pairs. Perhaps this is best possible. In fact, there are $n^{1/3}$ points at distance 1 from every other point. For every $0 < \alpha < 2$ there are n points so that for every point there are $\log^* n$ other points at distance α — again we do not know if this is best possible.

Purdy and I proved (using an idea of Kárteszi) that there are n points in the plane with no three on a line for which the unit distance occurs at least $cn \log n$ times. We have no nontrivial upper bound. If the points are in 3-space the unit distance can occur $n^{4/3}$ times (Hickerson, Pach and myself) but if we also assume that no four are on a plane, we can do no better than $cn \log n$.

Szekeres and I proved, that if $\binom{2n-4}{n-2} + 1$ points are given in a plane no three on a line, then we can always select among them n points which are the vertices of a convex n-gon. Probably $2^{n-2} + 1$ is the correct value — we proved that 2^{n-2} is not enough. This problem (which was due to E. Klein, i.e., Mrs. Szekeres) had a great influence.