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THE FUGLEDE–PUTNAM THEOREM AND PUTNAM'S INEQUALITY FOR QUASI-CLASS (A, k) OPERATORS

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ABSTRACT. An operator $T \in B(\mathcal{H})$ is called quasi-class (A, k) if $T^{*k}(|T^2| - |T|^2)T^k \geq 0$ for a positive integer k, which is a common generalization of class A. The famous Fuglede–Putnam's theorem is as follows: the operator equation $AX = XB$ implies $A^*X = XB^*$ when A and B are normal operators. In this paper, firstly we show that if X is a Hilbert-Schmidt operator, A is a quasi-class (A, k) operator and B^* is an invertible class A operator such that $AX = XB$, then $A^*X = XB^*$. Secondly we consider the Putnam's inequality for quasi-class (A, k) operators and we also show that quasisimilar quasi-class (A, k) operators have equal spectrum and essential spectrum.

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