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SINGULAR VALUE AND ARITHMETIC-GEOMETRIC MEAN INEQUALITIES FOR OPERATORS

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ABSTRACT. A singular value inequality for sums and products of Hilbert space operators is given. This inequality generalizes several recent singular value inequalities, and includes that if A, B, and X are positive operators on a complex Hilbert space H, then

$$s_j\left(A^{1/2}XB^{1/2}\right) \leq \frac{1}{2}\|X\| \; s_j\left(A+B\right), \;\; j=1,2,\cdots,$$

which is equivalent to

$$s_j\left(A^{1/2}XA^{1/2}-B^{1/2}XB^{1/2}\right) \leq \|X\|\, s_j\left(A\oplus B\right), \;\; j=1,2,\cdots.$$

Other singular value inequalities for sums and products of operators are presented. Related arithmetic–geometric mean inequalities are also discussed.

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