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ON RELATIONS AMONG SOLUTIONS OF THE HERMITIAN MATRIX EQUATION $AXA^* = B$ AND ITS THREE SMALL EQUATIONS

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Dedicated to Professor Tsuyoshi Ando for his significant contributions in matrix and operator theory

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ABSTRACT. Assume that the linear matrix equation $AXA^* = B = B^*$ has a Hermitian solution and is partitioned as $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} X \begin{bmatrix} A_1^* & A_2^* \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21}^* & B_{22} \end{bmatrix}$. We study in this paper relations among the Hermitian solutions of the equation and the three small-size matrix equations $A_1 X_1 A_1^* = B_{11}$, $A_1 X_2 A_2^* = B_{12}$ and $A_2 X_3 A_2^* = B_{22}$. In particular, we establish closed-form formulas for calculating the maximal and minimal ranks and inertias of $X - X_1 - X_2 - X_2^* - X_3$, and use the formulas to derive necessary and sufficient conditions for the Hermitian matrix equality $X = X_1 + X_2 + X_2^* + X_3$ to hold and Hermitian matrix inequalities $X > (\geq, <, \leq) X_1 + X_2 + X_2^* + X_3$ to hold in the Löwner partial ordering.

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