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ORTHOGONALLY ADDITIVE AND ORTHOGONALLY MULTIPLICATIVE HOLOMORPHIC FUNCTIONS OF MATRICES

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This paper is dedicated to Professor Tsuyoshi Ando

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ABSTRACT. Let $H : M_m \to M_m$ be a holomorphic function of the algebra M_m of complex $m \times m$ matrices. Suppose that H is orthogonally additive and orthogonally multiplicative on self-adjoint elements. We show that either the range of H consists of zero trace elements, or there is a scalar sequence $\{\lambda_n\}$ and an invertible S in M_m such that

$$H(x) = \sum_{n\geq 1} \lambda_n S^{-1} x^n S, \quad \forall x \in M_m, \text{ or } H(x) = \sum_{n\geq 1} \lambda_n S^{-1} (x^t)^n S, \quad \forall x \in M_m.$$

Here, x^t is the transpose of the matrix x. In the latter case, we always have the first representation form when H also preserves zero products. We also discuss the cases where the domain and the range carry different dimensions.

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