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APPROXIMATION PROBLEMS IN THE RIEMANNIAN METRIC ON POSITIVE DEFINITE MATRICES

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ABSTRACT. There has been considerable work on matrix approximation problems in the space of matrices with Euclidean and unitarily invariant norms. We initiate the study of approximation problems in the space $\mathbb{P}$ of all $n \times n$ positive definite matrices with the Riemannian metric δ_2. Our main theorem reduces the approximation problem in $\mathbb{P}$ to an approximation problem in the space of Hermitian matrices and then to that in $\mathbb{R}^n$. We find best approximants to positive definite matrices from special subsets of $\mathbb{P}$. The corresponding question in Finsler spaces is also addressed.

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