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EFFECTIVE H^∞ INTERPOLATION CONSTRAINED BY WEIGHTED HARDY AND BERGMAN NORMS

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ABSTRACT. Given a finite subset σ of the unit disc $\mathbb{D}$ and a holomorphic function f in $\mathbb{D}$ belonging to a class X, we are looking for a function g in another class Y which satisfies $g_{|\sigma} = f_{|\sigma}$ and is of minimal norm in Y. More precisely, we consider the interpolation constant $c(\sigma, X, Y) = \sup_{f\in X,\, \|f\|_X\leq 1} \inf_{g_{|\sigma}=f_{|\sigma}} \|g\|_Y$. When $Y = H^\infty$, our interpolation problem includes those of Nevanlinna–Pick and Carathéodory–Schur. If X is a Hilbert space belonging to the families of weighted Hardy and Bergman spaces, we obtain a sharp upper bound for the constant $c(\sigma, X, H^\infty)$ in terms of $n = \operatorname{card}\sigma$ and $r = \max_{\lambda\in\sigma} |\lambda| < 1$. If X is a general Hardy–Sobolev space or a general weighted Bergman space (not necessarily of Hilbert type), we also establish upper and lower bounds for $c(\sigma, X, H^\infty)$ but with some gaps between these bounds. This problem of constrained interpolation is partially motivated by applications in matrix analysis and in operator theory.

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