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ON SOME DIFFERENCE SEQUENCE SPACES OF WEIGHTED MEANS AND COMPACT OPERATORS

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ABSTRACT. In the peresent paper, by using generalized weighted mean and difference matrix of order m, we introduce the sequence spaces $X(u, v, \Delta^{(m)})$, where X is one of the spaces ℓ_∞, c or c_0. Also, we determine the α-, β- and γ-duals of those spaces and construct their Schauder bases for $X \in \{c, c_0\}$. Morever, we give the characterization of the matrix mappings on the spaces $X(u, v, \Delta^m)$ for $X \in \{\ell_\infty, c, c_0\}$. Finally, we characterize some classes of compact operators on the spaces $\ell_\infty(u, v, \Delta^m)$ and $c_0(u, v, \Delta^m)$ by using the Hausdorff measure of noncompactness.

1. Introduction

Let w be the space of real sequences. Any vector subspace of w is called as a sequence space. By ℓ_∞, c, c_0 and $\ell_p (1 < p < \infty)$, we denote the sequence spaces of all bounded, convergent, null sequences and p-absolutely convergent series, respectively. Also, we shall write ϕ for the set of all finite sequences that terminate in zeros, $e = (1, 1, 1, \cdots)$ and $e^{(n)}$ for the sequence whose only non-zero term 1 is at the nth place for each $n \in \mathbb{N}$, where $\mathbb{N} = \{0, 1, 2, \cdots\}$. Let $A = (a_{nk})$ be an infinite matrix of real numbers a_{nk} $(n, k \in \mathbb{N})$ and A_n denote the sequence in the nth row of A, that is $A_n = (a_{nk})_{k=0}^{\infty}$ for every $n \in \mathbb{N}$. In addition, if $x = (x_k) \in w$ then we define the A-transform of x as the sequence $Ax = \{(Ax)_n\}$, where

$$(Ax)_n = \sum_k a_{nk} x_k; \quad (n \in \mathbb{N}), \tag{1.1}$$

provided the series on the right converges for each $n \in \mathbb{N}$.

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Let X and Y be two sequence spaces. By (X, Y), we denote the class of all matrices A such that $A : X \to Y$. Thus, $A \in (X, Y)$ if and only if $Ax = \{(Ax)_n\}_{n\in\mathbb{N}} \in Y$ for all $x \in X$. Morever, the matrix domain X_A of an infinite matrix A in sequence space X is defined by

$$X_A = \{x = (x_k) \in w : Ax \in X\}. \tag{1.2}$$

The approach constructing a new sequence space by means of the matrix domain of a particular limitation method has recently been employed by several authors, see for instance [1]-[6],[8],[11]-[15],[19],[20],[26]-[29]. Also in the literature, there are many papers concerning the new sequence spaces derived by the domain of generalized weighted mean or the difference matrix order m(see [1, 2, 6, 12, 13, 15, 20, 22, 23, 28, 29]).

In the present paper, we define the new sequence spaces by using generalized weighted mean and difference matrix order m. Further, we determine the α-, β- and γ-duals of these spaces and construct their Schauder bases. Morever, we characterize some related matrix classes. Finally, by using the Hausdorff measure of noncompactness, we give the characterization of some classes of compact operators on these spaces.

2. The Sequence Spaces $X(u, v, \Delta^{(m)})$ for $X \in \{\ell_\infty, c, c_0\}$

In this section, we define the sequence spaces $\ell_\infty(u, v, \Delta^{(m)})$, $c(u, v, \Delta^{(m)})$ and $c_0(u, v, \Delta^{(m)})$ derived by the composition of the generalized weighted mean and difference matrix order m, and show that these spaces are the BK-spaces which are linearly isomorphic to the spaces ℓ_∞, c, c_0, respectively. Furthermore, we give the bases for the spaces $c(u, v, \Delta^{(m)})$ and $c_0(u, v, \Delta^{(m)})$.

If a normed space λ contains a sequence (b_n) with the property that for every $x \in \lambda$, there is a unique of scalars (α_n) such that

$$\lim_{n\to\infty} \|x - (\alpha_0 b_0 + \alpha_1 b_1 + \cdots + \alpha_n b_n)\| = 0,$$

then (b_n) is called a Schauder basis for λ. The series $\sum \alpha_k b_k$ which has the sum x is then called the expansion of x with respect to (b_n) and written as $x = \sum \alpha_k b_k$.

A sequence space X is called *FK space* if it is a complete linear metric space with continuous coordinates $p_n : X \to \mathbb{R}$ $(n \in \mathbb{N})$, where $\mathbb{R}$ denotes the real field and $p_n(x) = x_n$ for all $x = (x_k) \in X$ and every $n \in \mathbb{N}$. A *BK space* is a normed *FK space*, that is, a *BK space* is a Banach space with continuous coordinates. The space ℓ_p $(1 \le p < \infty)$ is BK space with $\|x\|_p = \left(\sum_{k=0}^{\infty} |x_k|^p\right)^{1/p}$ and c_0, c and ℓ_∞ are BK spaces with $\|x\|_\infty = \sup_k |x_k|$.

Let m denote a positive integer throughout and the operator $\Delta^{(m)} : w \to w$ be defined by

$$(\Delta^{(1)}x)_k = x_k - x_{k-1}, \quad (k = 0, 1, 2, \cdots),$$

$$\Delta^{(m)} = \Delta^{(1)} \text{ o } \Delta^{(m-1)} (m \ge 2).$$

We shall write $\Delta = \Delta^{(1)}$ for short and use the convention that any term with a negative subscript is equal to naught.

By U, we denote for the set of all sequences $u = (u_n)$ such that $u_n \neq 0$ for all $n \in \mathbb{N}$. For $u \in U$, let $1/u = (1/u_n)$. Let $u, v \in U$ and define the matrix $G(u,v) = (g_{nk})$ by

$$g_{nk} = \begin{cases} u_n v_k, & (0 \leq k \leq n) \\ 0, & (k > n) \end{cases}$$

for all $k, n \in \mathbb{N}$, where u_n depends only on n and v_k only on k. The matrix $G(u,v)$, defined above, is called as generalized weighted mean or factorable matrix [2].

Malkowsky and Savaş[20] have defined the sequence spaces $Z(u,v,X)$ which consists of all sequences such that $G(u,v)$-transforms of them are in $X \in \{\ell_\infty, c, c_0, \ell_p\}$. Başar and Altay[2] have examined the paranormed sequence spaces $\lambda(u,v;p)$ which are derived by the generalized weighted mean and proved that the spaces $\lambda(u,v;p)$ and $\lambda(p)$ are linearly isomorphic, where $\lambda(p)$ denotes the one of the sequence spaces $\ell_\infty(p)$, $c(p)$ and $c_0(p)$ defined by Maddox[14]. Recently, Polat, Karakaya and Şimsek [29] have studied the sequence spaces $\lambda(u,v,\Delta)$ which consists of all sequences such that $G(u,v,\Delta)$-forms of them are in $\lambda \in \{\ell_\infty, c, c_0\}$, where $G(u,v,\Delta) = G(u,v).\Delta$.

Following[20, 2, 29], we define the sequence spaces $X(u,v,\Delta^{(m)})$ for $X \in \{\ell_\infty, c, c_0\}$ by

$$X(u,v,\Delta^{(m)}) = \left\{x = (x_k) \in w : y = \left((G^{(m)}x)_k\right) \in X\right\},$$

where the sequence $y = (y_k)$ is the $G^{(m)} = G(u,v).\Delta^m$-transform of a sequence $x = (x_k)$, that is,

$$y_k = (G^{(m)}x)_k = u_k \sum_{j=0}^{k} \left[\sum_{i=j}^{k} \binom{m}{i-j} (-1)^{i-j} v_i\right] x_j; \quad (k \in \mathbb{N}). \tag{2.2}$$

With the notation of (1.2), we can redefine the spaces $X(u,v,\Delta^{(m)})$ for $X \in \{\ell_\infty, c, c_0\}$ as the matrix domains of the triangle $G^{(m)}$ in the spaces $X \in \{\ell_\infty, c, c_0\}$, that is

$$X(u,v,\Delta^{(m)}) = X_{G^{(m)}}. \tag{2.3}$$

The definition in (2.3) includes the following special cases:

(i) If $m = 1$, then $X(u,v,\Delta^{(m)}) = \lambda(u,v,\Delta)$ (cf[29, 22]).

(ii) If $v = (\lambda_k - \lambda_{k-1})$, $u = (1/\lambda_n)$, $m = 1$ and $X = c, c_0$, then $X(u,v,\Delta^{(m)}) = c_0^\lambda(\Delta)$, $c^\lambda(\Delta)$ (cf[26]).

(iii) If $v = (1 + r^k)$, $u = (1/(n+1))$, $m = 1$ and $X = c, c_0, \ell_\infty$, then $X(u,v,\Delta^{(m)}) = a_0^r(\Delta)$, $a_c^r(\Delta)$, $a_\infty^r(\Delta)$ (cf[3, 9, 10]).

Throughout we shall assume that the sequences $x = (x_k)$ and $y = (y_k)$ are connected by the relation (2.2), that is, y is the $G^{(m)}$-transform of x. Then, the sequence x is in any of the spaces $c_0(u,v,\Delta^{(m)})$, $c(u,v,\Delta^{(m)})$ or $\ell_\infty(u,v,\Delta^{(m)})$ if and only if y is in the respective one of the spaces c_0, c or ℓ_∞. In addition, one can easily derive that

$$x_k = \sum_{j=0}^{k} \sum_{i=j}^{j+1} \binom{m+k-i-1}{k-i} \frac{(-1)^{i-j}}{u_k v_i} y_j; \quad (k \in \mathbb{N}). \tag{2.4}$$

Now, we may begin with the following result which is essential in the text.

Theorem 2.1. *The sequence spaces $X(u,v,\Delta^{(m)})$ for $X \in \{\ell_\infty, c, c_0\}$ are Banach spaces with the norm given by*

$$\|x\|_{X(u,v,\Delta^{(m)})} = \|y\|_\infty = \sup_k \left| u_k \sum_{j=0}^{k} \left[\sum_{i=j}^{k} \binom{m}{i-j} (-1)^{i-j} v_i \right] x_j \right|. \tag{2.5}$$

Proof. Let X be any of the spaces c_0, c or ℓ_∞. Since it is a routine verification to show that $X(u,v,\Delta^{(m)})$ is a linear space with respect to coordinate-wise addition and scalar multiplication and is a normed space with the norm defined by (2.5) we omit the details. To prove the theorem, we show that every Cauchy sequence in $X(u,v,\Delta^{(m)})$ is convergent . Suppose $(x^{(n)})_{n=0}^{\infty}$ is a Cauchy sequence in $X(u,v,\Delta^{(m)})$. Thus, $(\forall \varepsilon > 0)(\exists N \in \mathbb{N})(\forall r, s \geq N)$;

$$(\left\|G^{(m)}x^{(r)} - G^{(m)}x^{(s)}\right\|_X = \left\|x^{(r)} - x^{(s)}\right\|_{X(u,v,\Delta^{(m)})} < \varepsilon).$$

So the sequence $(G^{(m)}x^{(n)})_{n=0}^{\infty}$ in X is Cauchy and since X is Banach, there exsits $x \in X$ such that

$$\left\|G^{(m)}x^{(n)} - x\right\|_X \to 0 \text{ as } n \to \infty.$$

But $x = (G^{(m)})\ (G^{(m)})^{-1}x$, so

$$\left\|G^{(m)}x^{(n)} - (G^{(m)})(G^{(m)})^{-1}x\right\|_X = \left\|x^{(n)} - (G^{(m)})^{-1}x\right\|_{X(u,v,\Delta^{(m)})} \to 0 \text{ as} n \to \infty.$$

Now, since $(G^{(m)})^{-1}x \in X(u,v,\Delta^{(m)})$ this completes the proof. □

Theorem 2.2. *Let X is any of the spaces c_0, c or ℓ_∞. Then the sequence space $X(u,v,\Delta^{(m)})$ is linearly isomorphic to the space X, that is $X(u,v,\Delta^{(m)}) \cong X$.*

Proof. Let

$$L : X(u,v,\Delta^{(m)}) \to X$$

defined by $L(x) = G^{(m)}x$. Since L is linear, bijective and norm preserving, we are done. □

Theorem 2.3. *Define the sequences $c^{(k)} = \left\{c_n^{(k)}\right\}_{n\in\mathbb{N}}$ and $c^{(-1)} = \left\{c_n^{(-1)}\right\}$ by*

$$c_n^{(k)} = \begin{cases} 0, & (n < k) \\ \sum_{j=k}^{k+1} \binom{m+n-j-1}{n-j} \frac{(-1)^{j-k}}{u_k v_j} & (n \geq k) \end{cases} ; \quad (n \in \mathbb{N}) \tag{2.6}$$

and

$$c_n^{(-1)} = \sum_{j=0}^{n} \sum_{i=j}^{j+1} \binom{m+n-i-1}{n-i} \frac{(-1)^{i-j}}{u_j v_i}; \quad (n \in \mathbb{N}).$$

a) Then, the sequence $\left(c^{(k)}\right)_{k=0}^{\infty}$ is a basis for the space $c_0(u,v,\Delta^{(m)})$ and every $x \in c_0(u,v,\Delta^{(m)})$ has a unique repsentation of the form

$$x = \sum_k (G^{(m)}x)_k c^{(k)}.$$

b) Then $\left(c^{(k)}\right)_{k=-1}^{\infty}$ *is a Schauder basis for* $c(u,v,\Delta^{(m)})$ *and every* $x \in c(u,v,\Delta^{(m)})$ *has a unique representation of the form*

$$x = lc^{(-1)} + \sum_k \left[(G^{(m)}x)_k - l\right] c^{(k)},$$

where $l = \lim_{k\to\infty}(G^{(m)}x)_k$.

Proof. This is an immediate consequence of [12, Lemma 2.3]. □

3. The $\alpha-, \beta-$ and $\gamma-$duals of the spaces $X(u,v,\Delta^{(m)})$ for $X \in \{\ell_\infty, c, c_0\}$

In the present section, we determine the $\alpha-, \beta-$ and $\gamma-$duals of the spaces $\ell_\infty(u,v,\Delta^{(m)}), c(u,v,\Delta^{(m)})$ and $c_0(u,v,\Delta^{(m)})$.

For the sequence spaces λ and μ, the set $S(\lambda,\mu)$ defined by

$$S(\lambda,\mu) = \{z = (z_k) \in w : xz = (x_k z_k) \in \mu \text{ for all } x \in \lambda\} \tag{3.1}$$

is called the multiplier space of λ and μ. With the notation (3.1), the $\alpha-, \beta-$ and $\gamma-$ duals of a sequence space λ, which are respectively denoted by λ^α, λ^β and λ^γ are, defined by

$$\lambda^\alpha = S(\lambda, \ell_1) \quad , \quad \lambda^\beta = S(\lambda, cs) \text{ and } \lambda^\gamma = S(\lambda, bs),$$

where ℓ_1, cs and bs are the spaces of all absolutely, convergent and bounded series, respectively.

Throughout, let $\mathcal{F}$ denote the collection of all nonempty and finite subsets of $\mathbb{N}$.

Now, we give the following lemmas (see [31]) which are needed in proving Theorems 3.3 - 3.5.

Lemma 3.1. $A \in (c_0,\ell_1) = (c,\ell_1) = (\ell_\infty,\ell_1)$ *if and only if*

$$\sup_{K\in\mathcal{F}} \sum_n \left|\sum_{k\in K} a_{nk}\right| < \infty.$$

Lemma 3.2. $A \in (c_0,\ell_\infty) = (c,\ell_\infty) = (\ell_\infty,\ell_\infty)$ *if and only if*

$$\sup_{n\in\mathbb{N}} \sum_k |a_{nk}| < \infty. \tag{3.2}$$

Now we prove the following results:

Theorem 3.3. *The α-dual of the spaces $X(u,v,\Delta^{(m)})$ for $X \in \{\ell_\infty, c, c_0\}$ is the set*

$$d_1 = \left\{a = (a_n) \in w : \sup_{K\in\mathcal{F}} \sum_n \left|\sum_{k\in K} c_{nk}\right| < \infty\right\},$$

where the matrix $C = (c_{nk})$ is defined via the sequence $a = (a_n)$ by

$$c_{nk} = \begin{cases} \sum_{j=k}^{k+1} \binom{m+n-j-1}{n-j} \frac{(-1)^{j-k}}{u_k v_j} a_n & (0 \le k \le n) \\ 0 & (k > n) \end{cases} ; (n,k \in \mathbb{N}).$$

Proof. Let $X \in \{\ell_\infty, c, c_0\}$ and $a = (a_n) \in w$. Then, by bearing in mind the relation (2.2) and (2.4), we immediately derive that

$$a_n x_n = \sum_{k=0}^{n} \left[\sum_{j=k}^{k+1} \binom{m+n-j-1}{n-j} \frac{(-1)^{j-k}}{u_k v_j} a_n \right] y_k; \quad (n \in \mathbb{N}). \tag{3.3}$$

Thus, we observe by (3.3) that $ax = (a_n x_n) \in \ell_1$ whenever $x = (x_k) \in X(u, v, \Delta^{(m)})$ if and only if $Cy \in \ell_1$ whenever $y = (y_k) \in X$. This means that the sequence $a = (a_n)$ is in the α-dual of the spaces $X(u, v, \Delta^{(m)})$ if and only if $C \in (X, \ell_1)$. We therefore obtain by Lemma 3.1 with C instead of A that $a \in \left\{X(u, v, \Delta^{(m)})\right\}^\alpha$ if and only if

$$\sup_{K \in \mathcal{F}} \sum_n \left| \sum_{k \in K} c_{nk} \right| < \infty$$

which leads us to the consequence that $\left\{X(u, v, \Delta^{(m)})\right\}^\alpha = d_1$. This concludes the proof. □

Now, let $x, y \in w$ be connected by the relation (2.2). Then, by using (2.4), we can easily derive that

$$\begin{aligned}
\sum_{k=0}^{n} a_k x_k &= \sum_{k=0}^{n} \left[\sum_{j=0}^{k} \sum_{i=j}^{j+1} \binom{m+k-i-1}{k-i} \frac{(-1)^{i-j}}{u_j v_i} y_j \right] a_k \\
&= \sum_{k=0}^{n} \left[\sum_{j=k}^{n} \left(\sum_{i=k}^{k+1} \binom{m+j-i-1}{j-i} \frac{(-1)^{i-k}}{u_k v_i} \right) a_j \right] y_k \\
&= \sum_{k=0}^{n} \left[\sum_{j=k}^{n} \nabla^{(m)}(j,k) a_j \right] y_k; \quad (n \in \mathbb{N}),
\end{aligned} \tag{3.4}$$

where

$$\nabla^{(m)}(j,k) = \sum_{i=k}^{k+1} \binom{m+j-i-1}{j-i} \frac{(-1)^{i-k}}{u_k v_i}. \tag{3.5}$$

This leads us to the following result:

Theorem 3.4. *Define the sets d_2, d_3, d_4 and d_5 as follows:*

$$d_2 = \left\{ a = (a_k) \in w : \sup_{n \in \mathbb{N}} \sum_{k=0}^{n} \left| \sum_{j=k}^{n} \nabla^{(m)}(j,k) a_j \right| < \infty \right\},$$

$$d_3 = \left\{ a = (a_k) \in w : \sum_{j=k}^{\infty} \nabla^{(m)}(j,k) a_j \text{ exists for each } k \in \mathbb{N} \right\},$$

$$d_4 = \left\{ a = (a_k) \in w : \lim_{n \to \infty} \sum_{k=0}^{n} \sum_{j=k}^{n} \nabla^{(m)}(j,k) a_j \quad \text{exists} \right\}$$

and

$$d_5 = \left\{ a = (a_k) \in w : \lim_{n\to\infty} \sum_k \left| \sum_{j=k}^{n} \nabla^{(m)}(j,k) a_j \right| = \sum_k \left| \sum_{j=k}^{\infty} \nabla^{(m)}(j,k) a_j \right| \right\}.$$

Then $\left\{c_0(u,v,\Delta^{(m)})\right\}^{\beta} = d_2 \cap d_3$, $\left\{c(u,v,\Delta^{(m)})\right\}^{\beta} = d_2 \cap d_3 \cap d_4$ *and* $\left\{\ell_\infty(u,v,\Delta^{(m)})\right\}^{\beta} = d_3 \cap d_5$.

Theorem 3.5. *The* γ*-dual of the spaces* $X(u,v,\Delta^{(m)})$ *for* $X \in \{\ell_\infty, c, c_0\}$ *is the set* d_2.

Proof. This result can be obtained from (3.2) in Lemma 3.2 by using (3.4). □

4. Certain matrix mappings on the spaces $X(u,v,\Delta^{(m)})$ for $X \in \{\ell_\infty, c, c_0\}$

In this section, we state some results which characterize various matrix mappings on the spaces $c_0(u,v,\Delta^{(m)})$, $c(u,v,\Delta^{(m)})$ and $\ell_\infty(u,v,\Delta^{(m)})$ and between them.

For an infinite matrix $A = (a_{nk})$, we shall write for brevity that

$$\bar{a}_{nk}^{\ell} = \sum_{j=k}^{\ell} \nabla^{(m)}(j,k) a_{nj}; \quad (k < m)$$

and

$$\bar{a}_{nk} = \sum_{j=k}^{\infty} \nabla^{(m)}(j,k) a_{nj} \tag{4.1}$$

for all $n, k, \ell \in \mathbb{N}$ provided the series on the right hand to be convergent. Further, let x, $y \in w$ be connected by the relation (2.2). Then, we have by (2.4) that

$$\sum_{k=0}^{\ell} a_{nk} x_k = \sum_{k=0}^{\ell} \bar{a}_{nk}^{\ell} y_k; \quad (n, \ell \in \mathbb{N}). \tag{4.2}$$

In particular, let $x \in c(u,v,\Delta^{(m)})$ and $A_n = (a_{nk})_{k=0}^{\infty} \in \left\{c(u,v,\Delta^{(m)})\right\}^{\beta}$ for all $n \in \mathbb{N}$. Then, we obtain, by passing to limit in (4.2) as $\ell \to \infty$ and using Theorem 3.4, that

$$\sum_{k=0}^{\infty} a_{nk} x_k = \sum_{k=0}^{\infty} \bar{a}_{nk} y_k; \quad (n \in \mathbb{N})$$

which gives the equality

$$\sum_{k=0}^{\infty} a_{nk} x_k = \sum_{k=0}^{\infty} \bar{a}_{nk} (y_k - l) + l \sum_{k=0}^{\infty} \bar{a}_{nk}; \quad (n \in \mathbb{N}), \tag{4.3}$$

where $l = \lim_{k\to\infty} y_k$.

Now, let us consider the following conditions:

$$\sup_n \left(\sum_{k=0}^{\infty} |\bar{a}_{nk}| \right) < \infty, \tag{4.4}$$

$$\lim_{\ell\to\infty}\sum_{k=0}^{\infty}\left|\bar{a}_{nk}^{\ell}\right| = \sum_{k=0}^{\infty}\left|\bar{a}_{nk}\right|;\ (n\in\mathbb{N}), \tag{4.5}$$

$$\bar{a}_{nk} \text{ exists for all } k,n\in\mathbb{N}, \tag{4.6}$$

$$\sup_{\ell\in\mathbb{N}}\sum_{k=0}^{\ell}\left|\bar{a}_{nk}^{\ell}\right| < \infty;\ \ (n\in\mathbb{N}), \tag{4.7}$$

$$\lim_{n\to\infty}\bar{a}_{nk} = \bar{\alpha}_k;\ \ (k\in\mathbb{N}), \tag{4.8}$$

$$\lim_{n\to\infty}\sum_{k=0}^{\infty}\left|\bar{a}_{nk}-\bar{\alpha}_k\right| = 0, \tag{4.9}$$

$$\lim_{n\to\infty}\sum_{k=0}^{\infty}\bar{a}_{nk} = \alpha, \tag{4.10}$$

$$\sum_{k=0}^{\infty}\bar{a}_{nk} \text{ converges for all } n\in\mathbb{N}, \tag{4.11}$$

$$\lim_{n\to\infty}\sum_{k=0}^{\infty}\left|\bar{a}_{nk}\right| = 0, \tag{4.12}$$

$$\lim_{n\to\infty}\bar{a}_{nk} = 0 \text{ for all } k\in\mathbb{N}, \tag{4.13}$$

$$\sup_{K\in\mathcal{F}}\left(\sum_{n=0}^{\infty}\left|\sum_{k\in K}\bar{a}_{nk}\right|^{p}\right) < \infty;\ \ (1\le p<\infty), \tag{4.14}$$

$$\lim_{n\to\infty}\sum_{k=0}^{\infty}\bar{a}_{nk} = 0. \tag{4.15}$$

Then, by combining Theorem 3.4 with the results of Stieglitz and Tietz[31], we immediately derive the following results by using (4.3).

Theorem 4.1. *We have*
(a) $A\in(\ell_\infty(u,v,\Delta^{(m)}),\ell_\infty)$ *if and only if* (4.4), (4.5) *and* (4.6) *hold.*
(b) $A\in(c(u,v,\Delta^{(m)}),\ell_\infty)$ *if and only if* (4.4), (4.6) *and* (4.7) *hold.*
(c) $A\in(c_0(u,v,\Delta^{(m)}),\ell_\infty)$ *if and only if* (4.4), (4.6) *and* (4.7) *hold.*

Theorem 4.2. *We have*
(a) $A\in(\ell_\infty(u,v,\Delta^{(m)}),c)$ *if and only if* (4.5), (4.6), (4.8) *and* (4.9) *hold.*
(b) $A\in(c(u,v,\Delta^{(m)}),c)$ *if and only if* (4.4), (4.6), (4.7), (4.8) *and* (4.10) *hold.*
(c) $A\in(c_0(u,v,\Delta^{(m)}),c)$ *if and only if* (4.4), (4.6), (4.7) *and* (4.8) *hold.*

Theorem 4.3. *We have*
(a) $A\in(\ell_\infty(u,v,\Delta^{(m)}),c_0)$ *if and only if* (4.5), (4.6) *and* (4.12) *hold.*
(b) $A\in(c(u,v,\Delta^{(m)}),c_0)$ *if and only if* (4.4), (4.6), (4.7), (4.13) *and* (4.15) *hold.*
(c) $A\in(c_0(u,v,\Delta^{(m)}),c_0)$ *if and only if* (4.4), (4.6), (4.7) *and* (4.13) *hold.*

Theorem 4.4. *Let $1 \leq p < \infty$. Then, we have*
(a) $A \in (\ell_\infty(u,v,\Delta^{(m)}), \ell_p)$ if and only if (4.5), (4.6) and (4.14) hold.
(b) $A \in (c(u,v,\Delta^{(m)}), \ell_p)$ if and only if (4.6), (4.7), (4.11) and (4.14) hold.
(c) $A \in (c_0(u,v,\Delta^{(m)}), \ell_p)$ if and only if (4.6), (4.7) and (4.14) hold.

Now, we may present the lemma given by Başar and Altay [4, Lemma 5.3] which is useful for obtaining the characterization of some new matrix classes from Theorems 4.1-4.3.

Lemma 4.5. *Let λ, μ be any two sequence spaces, A be an infinite matrix and B a triangle matrix. Then, $A \in (\lambda, \mu_B)$ if and only if $T = BA \in (\lambda, \mu)$.*

We should finally note that, if a_{nk} is replaced by $t_{nk} = u_n \sum_{j=0}^{n} \left[\sum_{i=j}^{n} \binom{m}{i-j}(-1)^{i-j} v_i\right] a_{jk}$ for all $k, n \in \mathbb{N}$ in Theorems 4.1-4.3, then one can derive the characterization of the classes $(\lambda((u,v,\Delta^{(m)}), \mu(u,v,\Delta^{(m)}))$ from Lemma 4.5 with $B = G^{(m)}$, where $\lambda, \mu \in \{c_0, c, \ell_\infty\}$.

5. Measure of noncompactness of matrix operators on the sequence spaces $c_0(u,v,\Delta^{(m)})$ and $\ell_\infty(u,v,\Delta^{(m)})$

In this section, we characterize some classes of compact operators on the spaces $c_0(u,v,\Delta^{(m)})$ and $\ell_\infty(u,v,\Delta^{(m)})$ by using the Hausdorff measure of noncompactness.

It is quite natural to find conditions for a matrix map between BK-spaces to define a compact operator since a matrix transformation between BK-spaces are continuous. This can be achieved by applying the Hausdorff measure of noncompactness. In past, several authors characterized classes of compact operators given by infinite matrices on the some sequence spaces by using this method. For example see [5],[7]-[10],[12],[13],[18],[19],[21]-[23],[25],[30]. Recently, Malkowsky and Rakočević [17], Djolović and Malkowsky [11] and Mursaleen and Noman [24] established some identities or estimates for the operator norms and Hausdorff measures of noncompactness of linear operators given by infinite matrices that map an arbitrary BK-space or the matrix domains of triangles in arbitrary BK-spaces.

Let X be a normed space. Then, we write S_X for the unit sphere in X, that is, $S_X = \{x \in X : \|x\| = 1\}$. If X and Y be Banach spaces then $B(X,Y)$ is the set of all continuous linear operators $L : X \to Y$; $B(X,Y)$ is a Banach space with the operator norm defined by $\|L\| = \sup\{\|Lx\| : \|x\| \leq 1\}$ for all $L \in B(X,Y)$.

If $(X, \|.\|)$ is a normed sequence space, then we write

$$\|a\|_X^* = \sup_{x \in S_X} \left| \sum_{k=0}^{\infty} a_k x_k \right| \tag{5.1}$$

for $a \in w$ provided the expression on the right hand side exists and is finite which is the case whenever X is a BK space and $a \in X^\beta$ [32, p.107].

We recall that if X and Y are Banach spaces and L is a linear operator from X to Y, then L is said to be compact if its domain is all of X and for every bounded sequence (x_n) in X, the sequence $((Lx)_n)$ has a convergent subsequence in Y. We denote the class of such operators by $K(X,Y)$.

If (X,d) is a metric space, we write M_X for the class of all bounded subsets of X. By $B(x,r)=\{y\in X : d(x,y)<r\}$ we denote the open ball of radius $r>0$ with centre in x. Then the Hausdorff measure of noncompactness of the set $Q\in M_X$, denoted by $\chi(Q)$, is given by

$$\chi(Q)=\inf\left\{\varepsilon>0 : Q\subset\bigcup_{i=0}^{n}B(x_i,r_i),\ x_i\in X,\ r_i<\varepsilon(i=0,1,\cdots,n),\, n\in\mathbb{N}\right\}.$$

The function $\chi : M_X\to[0,\infty)$ is called the Hausdorff measure of noncompactness.

The basic properties of the Hausdorff measure of noncompactness can be found in [16], for example if Q, Q_1 and Q_2 are bounded subsets of a metric space (X,d), then

$$\chi(Q)=0 \text{ if and only if } Q \text{ is totally bounded,}$$
$$Q_1\subset Q_2 \text{ implies } \chi(Q_1)\le\chi(Q_2).$$

Further if X is a normed space, the function χ has some additional properties connected with the linear structure, e.g.

$$\chi(Q_1+Q_2)\le\chi(Q_1)+\chi(Q_2),$$
$$\chi(\alpha Q)=|\alpha|\,\chi(Q) \text{ for all } \alpha\in\mathbb{C},$$

where $\mathbb{C}$ is the complex field.

We shall need the following known results for our investigation.

Lemma 5.1. [22, Lemma 3.1]. *Let X denotes any of the spaces c_0 and ℓ_∞. If $A\in(X,c)$, then we have*

$$\alpha_k=\lim_{n\to\infty}a_{nk} \text{ exists for every } k\in\mathbb{N},$$
$$\alpha=(\alpha_k)\in\ell_1,$$
$$\sup_n\left(\sum_{k=0}^{\infty}|a_{nk}-\alpha_k|\right)<\infty,$$
$$\lim_{n\to\infty}(Ax)_n=\sum_{k=0}^{\infty}\alpha_k x_k \text{ for all } x=(x_k)\in X.$$

Lemma 5.2. [22, Lemma 1.1]. *Let X denotes any of the spaces c_0, c or ℓ_∞. Then, we have $X^\beta=\ell_1$ and $\|a\|_X^*=\|a\|_{\ell_1}$ for all $a\in\ell_1$.*

Lemma 5.3. [32, Theorem 4.2.8]. *Let X and Y be BK-spaces. Then we have $(X,Y)\subset B(X,Y)$, that is, every $A\in(X,Y)$ defines a linear operator $L_A\in B(X,Y)$, where $L_A(x)=Ax$ for all $x\in X$.*

Lemma 5.4. [12, Lemma 5.2]. *Let $X\supset\phi$ be BK-space and Y be any of the spaces c_0, c or ℓ_∞. If $A\in(X,Y)$, then*

$$\|L_A\|=\|A\|_{(X,\ell_\infty)}=\sup_n\|A_n\|_X^*<\infty.$$

Lemma 5.5. [22, Lemma 1.5]. *Let $Q \in M_{c_0}$ and $P_r : c_0 \to c_0$ $(r \in \mathbb{N})$ be the operator defined by $P_r(x) = (x_0, x_1, \cdots, x_r, 0, 0, \cdots)$ for all $x = (x_k) \in c_0$. Then, we have*

$$\chi(Q) = \lim_{r\to\infty} \left(\sup_{x\in Q} \|(I - P_r)(x)\|_{\ell_\infty} \right),$$

where I is the identity operator on c_0.

Further, we know by [16, Theorem 1.10] that every $z = (z_n) \in c$ has a unique repsentation $z = \bar{z}e + \sum_{n=0}^{\infty}(z_n - \bar{z})e^{(n)}$, where $\bar{z} = \lim_{n\to\infty} z_n$. Thus, we define the projectors $P_r : c \to c$ $(r \in \mathbb{N})$ by

$$P_r(z) = \bar{z}e + \sum_{n=0}^{r}(z_n - \bar{z})e^{(n)}; \quad (r \in \mathbb{N}) \tag{5.2}$$

for all $z = (z_n) \in c$ with $\bar{z} = \lim_{n\to\infty} z_n$. In this sitation, the following result gives an estimate for the Hausdorff measure of noncompactness in the BK space c.

Lemma 5.6. [22, Lemma 1.6]. *Let $Q \in M_c$ and $P_r : c \to c$ $(r \in \mathbb{N})$ be the projector onto the linear span of $\left\{e, e^{(0)}, e^{(1)}, \cdots, e^{(r)}\right\}$. Then, we have*

$$\frac{1}{2}.\lim_{r\to\infty} \left(\sup_{x\in Q} \|(I - P_r)(x)\|_{\ell_\infty} \right) \le \chi(Q) \le \lim_{r\to\infty} \left(\sup_{x\in Q} \|(I - P_r)(x)\|_{\ell_\infty} \right),$$

where I is the identity operator on c.

The next lemma is related to the Hausdorff measure of noncompactness of a bounded linear operator.

Lemma 5.7. [16, Thereom 2.25, Corollary 2.26]. *Let X and Y be Banach spaces and $L \in B(X, Y)$. Then we have*

$$\|L\|_\chi = \chi(L(S_X)) \tag{5.3}$$

and

$$L \in K(X, Y) \text{ if and only if } \|L\|_\chi = 0. \tag{5.4}$$

The following results will be needed in establishing our results.

Lemma 5.8. *Let X denotes any of the spaces $c_0(u, v, \Delta^{(m)})$ or $\ell_\infty(u, v, \Delta^{(m)})$. If $a = (a_k) \in X^\beta$ then $\bar{a} = (\bar{a}_k) \in \ell_1$ and the equality*

$$\sum_{k=0}^{\infty} a_k x_k = \sum_{k=0}^{\infty} \bar{a}_k y_k \tag{5.5}$$

holds for every $x = (x_k) \in X$, where $y = G^{(m)}x$ is the associated sequence defined by (2.2) and

$$\bar{a}_k = \sum_{j=k}^{\infty} \nabla^{(m)}(j, k) a_j; \quad (k \in \mathbb{N}).$$

Proof. This follows immediately by [26, Theorem 5.6]. □

Lemma 5.9. *Let X denotes any of the spaces $c_0(u, v, \Delta^{(m)})$ or $\ell_\infty(u, v, \Delta^{(m)})$. Then, we have*

$$\|a\|_X^* = \|\bar{a}\|_{\ell_1} = \sum_{k=0}^{\infty} |\bar{a}_k| < \infty$$

for all $a = (a_k) \in X^\beta$, where $\bar{a} = (\bar{a}_k)$ is as in Lemma 5.8.

Proof. Let Y be the respective one of the spaces c_0 or ℓ_∞, and take any $a = (a_k) \in X^\beta$. Then, we have by Lemma 5.8 that $\bar{a} = (\bar{a}_k) \in \ell_1$ and the equality (5.5) holds for all sequences $x = (x_k) \in X$ and $y = (y_k) \in Y$ which are connected by the relation (2.2). Further, it follows by (2.5) that $x \in S_X$ if and only if $y \in S_Y$. Thefore, we derive from (5.1) and (5.5) that

$$\|a\|_X^* = \sup_{x \in S_X} \left| \sum_{k=0}^{\infty} a_k x_k \right| = \sup_{y \in S_Y} \left| \sum_{k=0}^{\infty} \bar{a}_k y_k \right| = \|\bar{a}\|_Y^*$$

and since $\bar{a} \in \ell_1$, we obtain from Lemma 5.2 that

$$\|a\|_X^* = \|\bar{a}\|_Y^* = \|\bar{a}\|_{\ell_1} < \infty$$

which concludes the proof. □

Lemma 5.10. *Let X be any of the spaces $c_0(u, v, \Delta^{(m)})$ or $\ell_\infty(u, v, \Delta^{(m)})$, Y the respective one of the spaces c_0 or ℓ_∞, Z a sequence space and $A = (a_{nk})$ an infinite matrix. If $A \in (X, Z)$, then $\bar{A} \in (Y, Z)$ such that $Ax = \bar{A}y$ for all sequences $x \in X$ and $y \in Y$ which are connected by the relation (2.2), where $\bar{A} = (\bar{a}_{nk})$ is the associated matrix defined as in (4.1).*

Proof. This is immediate by [22, Lelmma 2.3]. □

Now, let $A = (a_{nk})$ be an infinite matrix and $\bar{A} = (\bar{a}_{nk})$ the associated matrix defined by (4.1). Then, we have the following result.

Theorem 5.11. *Let X denotes any of the spaces $c_0(u, v, \Delta^{(m)})$ or $\ell_\infty(u, v, \Delta^{(m)})$. Then, we have*

(a) If $A \in (X, c_0)$, then

$$\|L_A\|_\chi = \limsup_{n\to\infty} \sum_{k=0}^{\infty} |\bar{a}_{nk}| . \tag{5.6}$$

(b) If $A \in (X, c)$, then

$$\frac{1}{2} . \limsup_{n\to\infty} \sum_{k=0}^{\infty} |\bar{a}_{nk} - \bar{a}_k| \le \|L_A\|_\chi \le \limsup_{n\to\infty} \sum_{k=0}^{\infty} |\bar{a}_{nk} - \overline{\alpha}_k| , \tag{5.7}$$

where $\overline{a}_k$ is defined as in (4.8) for all $k \in \mathbb{N}$.

(c) If $A \in (X, \ell_\infty)$, then

$$0 \le \|L_A\|_\chi \le \limsup_{n\to\infty} \sum_{k=0}^{\infty} |\bar{a}_{nk}| . \tag{5.8}$$

Proof. Let us remark that the expressions in (5.6) and (5.8) exist by Theorems 4.3 and 4.1. Also, by combining Lemmas 5.1 and 5.10, we deduce that the expression in (5.7) exists.

We write $S = S_X$, for short. Then, we obtain by (5.3) and Lemma 5.3 that

$$\|L_A\|_\chi = \chi(AS). \tag{5.9}$$

For (a), we have $AS \in M_{c_0}$. Thus, it follows by applying Lemma 5.5 that

$$\chi(AS) = \lim_{r\to\infty}\left(\sup_{x\in S}\|(I-P_r)(Ax)\|_{\ell_\infty}\right), \tag{5.10}$$

where $P_r : c_0 \to c_0$ $(r \in \mathbb{N})$ is the operator defined by $P_r(x) = (x_0, x_1, \cdots, x_r, 0, 0, \cdots)$ for all $x = (x_k) \in c_0$. This yields that $\|(I-P_r)(Ax)\|_{\ell_\infty} = \sup_{n>r}|(Ax)_n|$ for all $x \in X$ and every $r \in \mathbb{N}$. Therefore, by using (1.1), (5.1) and Lemma 5.9, we have for every $r \in \mathbb{N}$ that

$$\sup_{x\in S}\|(I-P_r)(Ax)\|_{\ell_\infty} = \sup_{n>r}\|A_n\|_X^* = \sup_{n>r}\left\|\bar{A}_n\right\|_{\ell_1}.$$

This and (5.10) imply that

$$\chi(AS) = \lim_{r\to\infty}\left(\sup_{n>r}\left\|\bar{A}_n\right\|_{\ell_1}\right) = \limsup_{n\to\infty}\left\|\bar{A}_n\right\|_{\ell_1}.$$

Hence, we obtain that (5.6) from (5.9).

To prove (b), we have $AS \in M_c$. Thus, we are going to apply Lemma 5.6 to get an estimate for the value of $\chi(AS)$ in (5.9). For this, let $P_r : c \to c$ $(r \in \mathbb{N})$ be the projectors defined by (5.2). Then, we have for every $r \in \mathbb{N}$ that $(I-P_r)(z) = \sum_{n=r+1}^{\infty}(z_n - \bar{z})e^{(n)}$ and hence,

$$\|(I-P_r)(z)\|_{\ell_\infty} = \sup_{n>r}|z_n - \bar{z}| \tag{5.11}$$

for all $z = (z_n) \in c$ and every $r \in \mathbb{N}$, where $\bar{z} = \lim_{n\to\infty} z_n$ and I is identity operatoron c.

Now, by using (5.9) we obtain by applying Lemma 5.6 that

$$\frac{1}{2}\cdot\lim_{r\to\infty}\left(\sup_{x\in S}\|(I-P_r)(Ax)\|_{\ell_\infty}\right) \le \|L_A\|_\chi \le \lim_{r\to\infty}\left(\sup_{x\in S}\|(I-P_r)(Ax)\|_{\ell_\infty}\right). \tag{5.12}$$

On the other hand, it is given that $X = c_0(u, v, \Delta^{(m)})$ or $\ell_\infty(u, v, \Delta^{(m)})$, and let Y be the respective one of the spaces c_0 or ℓ_∞. Also, for every given $x \in X$, let $y \in Y$ be the associated sequence defined by (2.2). Since, $A \in (X, c)$, we have by Lemma 5.10 that $\bar{A} \in (Y, c)$ and $Ax = \bar{A}y$. Further, it follows from Lemma 5.1 that the limits $\overline{\alpha}_k = \lim_{n\to\infty}\overline{a}_{nk}$ exists for all k, $(\overline{\alpha}_k) \in \ell_1 = Y^\beta$ and

$\lim_{n\to\infty}(\bar{A}y)_n = \sum_{k=0}^{\infty}\overline{\alpha}_k y_k$. Consequently, we derive from (5.11) that

$$\begin{aligned}\|(I-P_r)(Ax)\|_{\ell_\infty} &= \left\|(I-P_r)(\bar{A}y)\right\|_{\ell_\infty}\\ &= \sup_{n>r}\left|(\bar{A}y)_n - \sum_{k=0}^{\infty}\overline{\alpha}_k y_k\right|\\ &= \sup_{n>r}\left|\sum_{k=0}^{\infty}(\overline{a}_{nk}-\overline{\alpha}_k)y_k\right|\end{aligned}$$

for all $r \in \mathbb{N}$. Moreover, since $x \in S = S_X$ if and only if $y \in S_Y$ we obtain by (5.1) and Lemma 5.2 that

$$\begin{aligned}\|(I-P_r)(Ax)\|_{\ell_\infty} &= \sup_{n>r}\left(\sup_{y\in S_Y}\left|\sum_{k=0}^{\infty}(\overline{a}_{nk}-\overline{\alpha}_k)y_k\right|\right)\\ &= \sup_{n>r}\left\|\bar{A}_n - \bar{\alpha}\right\|_Y^*\\ &= \sup_{n>r}\left\|\bar{A}_n - \bar{\alpha}\right\|_{\ell_1}\end{aligned}$$

for all $r \in \mathbb{N}$. Thus, we get (5.7) from (5.12).

Finally, to prove (c) we define the projectors $P_r : \ell_\infty \to \ell_\infty$ $(r \in \mathbb{N})$ as in the proof of part (a) for all $x = (x_k) \in \ell_\infty$. Then, we have

$$AS \subset P_r(AS) + (I-P_r)(AS); \quad (r \in \mathbb{N}).$$

Thus, it follows by the elementary properties of the function χ that

$$\begin{aligned}0 &\le \chi(AS) \le \chi(P_r(AS)) + \chi((I-P_r)(AS))\\ &= \chi((I-P_r)(AS)) \le \sup_{x\in S}\|(I-P_r)(Ax)\|_{\ell_\infty}\\ &= \sup_{n>r}\left\|\bar{A}_n\right\|_{\ell_1}\end{aligned}$$

for all $r \in \mathbb{N}$ and hence,

$$0 \le \chi(AS) \le \lim_{r\to\infty}\left(\sup_{n>r}\left\|\bar{A}_n\right\|_{\ell_1}\right) = \limsup_{r\to\infty}\left\|\bar{A}_n\right\|_{\ell_1}.$$

This and (5.9) together imply (5.8) and complete the proof. □

Corollary 5.12. *Let X denotes any of the spaces $c_0(u,v,\Delta^{(m)})$ or $\ell_\infty(u,v,\Delta^{(m)})$. Then, we have*

(a) If $A \in (X, c_0)$, then

$$L_A \textit{ is compact if and only if } \lim_{n\to\infty}\sum_{k=0}^{\infty}|\overline{a}_{nk}| = 0.$$

(b) If $A \in (X, c)$, then

$$L_A \textit{ is compact if and only if } \lim_{n\to\infty}\sum_{k=0}^{\infty}|\overline{a}_{nk}-\overline{\alpha}_k| = 0.$$

(c) If $A \in (X, \ell_\infty)$, then

$$L_A \text{ is compact if } \lim_{n\to\infty} \sum_{k=0}^{\infty} |\overline{a}_{nk}| = 0.$$

Proof. This result follows from Theorem 5.11 by using (5.4). □

Finally, we have the following observation.

Corollary 5.13. *For every matrix $A \in (\ell_\infty(u, v, \Delta^{(m)}), c_0)$ or $A \in (\ell_\infty(u, v, \Delta^{(m)}), c)$, the operator L_A is compact.*

Proof. Let $A \in (\ell_\infty(u, v, \Delta^{(m)}), c_0)$. Then we have by Theorem 4.3(a) that $\lim_{n\to\infty} (\sum_{k=0}^{\infty} |\overline{a}_{nk}|) = 0$. This leads us with Corollary 5.12(a) to the consequence that L_A is compact. Similarly, If $A \in (\ell_\infty(u, v, \Delta^{(m)}), c)$ then, from Theorem 4.2(a), we have that $\lim_{n\to\infty} (\sum_{k=0}^{\infty} |\overline{a}_{nk} - \overline{\alpha}_k|) = 0$, where $\overline{\alpha}_k = \lim_{n\to\infty} \overline{a}_{nk}$ for all k. Hence, we deduce from Corollary 5.12(b) that L_A is compact. □

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