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EIGENVALUE PROBLEM FOR A CLASS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper, we study eigenvalue problem for a class of nonlinear fractional differential equations

$$\begin{aligned} D_{0+}^{\alpha} u(t) &= \lambda f(u(t)), \quad 0 < t < 1, \\ u(0) = u(1) &= u'(0) = u'(1) = 0, \end{aligned}$$

where $3 < \alpha \leq 4$ is a real number, D_{0+}^{α} is the Riemann–Liouville fractional derivative, λ is a positive parameter and $f : (0, +\infty) \rightarrow (0, +\infty)$ is continuous. By the properties of the Green function and Guo–Krasnosel’skii fixed point theorem on cones, the eigenvalue intervals of the nonlinear fractional differential equation boundary value problem are considered, some sufficient conditions for the nonexistence and existence of at least one or two positive solutions for the boundary value problem are established. As an application, some examples are presented to illustrate the main results.

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