



Ann. Funct. Anal. 4 (2013), no. 2, 1–11

ANNALS OF FUNCTIONAL ANALYSIS

ISSN: 2008-8752 (electronic)

URL: [www.emis.de/journals/AFA/](http://www.emis.de/journals/AFA/)

## ON THE CESÁRO OPERATOR IN WEIGHTED $\ell^2$ -SEQUENCE SPACES AND THE GENERALIZED CONCEPT OF NORMALITY

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Communicated by D. H. Leung

ABSTRACT. The weighted Cesáro operator  $C_h$  in  $\ell^2(h)$ -spaces is investigated in terms of several concepts of normality, where  $h$  denotes a positive discrete measure on  $\mathbb{N}_0$ . We classify exactly those  $h$  for which  $C_h$  is hyponormal. Two examples related to the Haar measures of orthogonal polynomials are discussed. We show that the Cesáro operator is not always paranormal. Furthermore, we prove that the Cesáro operator is not quasinormal for any choice of  $h$ .

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*Date:* Received: 15 October 2012; Accepted: 6 November 2012.

*2010 Mathematics Subject Classification.* Primary 47B37; Secondary 47B20, 33D45.

*Key words and phrases.* Cesáro operator, quasinormal operator, hyponormal operator, paranormal operator, orthogonal polynomials.