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A STRONG VERSION OF THE BIRKHOFF–JAMES ORTHOGONALITY IN HILBERT C^* -MODULES

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This paper is dedicated to Professor T. Ando

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ABSTRACT. In this paper we introduce a strong version of the Birkhoff–James orthogonality in Hilbert C^* -modules. More precisely, we consider elements x and y of a Hilbert C^* -module V over a C^* -algebra \mathcal{A} which satisfy $\|x\| \leq \|x + ya\|$ for all $a \in \mathcal{A}$. We show that this relation can be described as the Birkhoff–James orthogonality of appropriate elements of V , and characterized in terms of states acting on the underlying C^* -algebra \mathcal{A} . Some analogous relations of this type are considered as well.

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