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Line-hyperline pairs of projective spaces and fundamental subgroups of linear groups*

Ralf Gramlich

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Abstract. This article provides an almost self-contained, purely combinatorial local recognition of the graph on the non-intersecting line-hyperline pairs of the projective space $\mathbb{P}_n(\mathbb{F})$ for $n \geqslant 8$ and $\mathbb{F}$ a division ring with the exception of the case $n = 8$ and $\mathbb{F} = \mathbb{F}_2$. Consequences of that result are a characterization of the hyperbolic root group geometry of $\mathrm{SL}_{n+1}(\mathbb{F})$, $\mathbb{F}$ a division ring, and a local recognition of certain groups containing a central extension of $\mathrm{PSL}_{n+1}(\mathbb{F})$, $\mathbb{F}$ a field, using centralizers of p-elements.

1 Introduction and preliminaries

The characterization of graphs and geometries using certain configurations that do or do not occur in some graph or geometry is a central problem in synthetic geometry. One class of such characterizations are the so-called local recognition theorems of locally homogeneous graphs. A graph Γ is called *locally homogeneous* if $\Gamma(x) \cong \Gamma(y)$ for all vertices $x, y \in \Gamma$, where $\Gamma(x)$ denotes the induced subgraph on the neighbours of x in Γ. A locally homogeneous graph Γ with $\Gamma(x) \cong \Delta$ is also called *locally* Δ. For some fixed graph Δ it is a natural question to ask for a classification of all connected graphs Γ that are locally Δ. A connected locally Δ graph Γ is *locally recognizable* if, up to isomorphism, Γ is the unique graph with that property. Several local recognition results of a lot of classes of graphs can be found in the literature. As an example we refer to the local recognition of the Kneser graphs by Jonathan I. Hall [7]; the Kneser graphs can be considered as 'thin' analogues of the graphs that are studied in this paper.

The present article focuses on graphs on line-hyperline pairs of projective spaces; more precisely, let $\mathbf{L}_n(\mathbb{F})$ denote the graph on the non-intersecting line-hyperline pairs of the projective space $\mathbb{P}_n(\mathbb{F})$ (where n is a natural number and $\mathbb{F}$ a division ring) in which two vertices are adjacent if the line of one vertex is contained in the hyperline of the other vertex and vice versa. Then the following holds.

* The present article was written while the author was a PhD student at TU Eindhoven.