EQUILIBRIUM STABILITY OF A SERVO ACTUATING FLIGHT CONTROLS IN A SERVOELASTIC FRAMEWORK

Ioan Ursu, Felicia Ursu, Andrei Halanay, Carmen Anca Safta

Abstract. The mounting structure stiffness's effects on mecanohydraulic servomechanisms actuating aicraft's primary flight controls are modeled as a system of ordinary differential equations. Stability analysis of equilibria reveals the presence of a critical case that is handled through the use of the Lyapunov-Malkin theorem. Stability charts are drawn using the Routh-Hurwitz criterion for stability polynomial and it is shown that the stability of the system can be ensured exploiting positive influence of structural feedback.

2000 *Mathematics Subject Classification*: 34H05, 34D20.

1. Introduction

Due to complex aeroservoelastic interaction between rigid body dynamics, structural dynamics, unsteady aerodynamics and hydraulic servos, a real problem of control design improvement permanently exists in the field [1], of airframe control. To address it, a control synthesis for an mechanohydraulic servoactuator (MHS) in a particular servoelastic framework defined by its finite mounting structure stiffness is developed in the present paper. The premise of the approach starts from a simple conjecture: as an aeroservoelastic system's component, asymptotically stable in the presence of exogeneous inputs, the hydraulic servoactuator will thus contribute to general stability and performance of flight control system. From this perspective, and extending recent results of the authors ([2] - [4]), the present work offers a model of treatment of the aeroservoelastic problems even in an early stage of system design.

2. The mathematical model

In Figure 1, a typical physical model of an MHS in the servoelastic framework is shown, defined by considering the mounting structure stiffness E. MHS is in fact a combination spool valve (SV)-hydrocylinder (HC) with piston. The first equation of the SV-controlled piston is obtained based on the fact that the flow into and out of the cylinder is described by two components, one being due to the movement of the piston and one to compressibility effects: for the case $\sigma > 0$

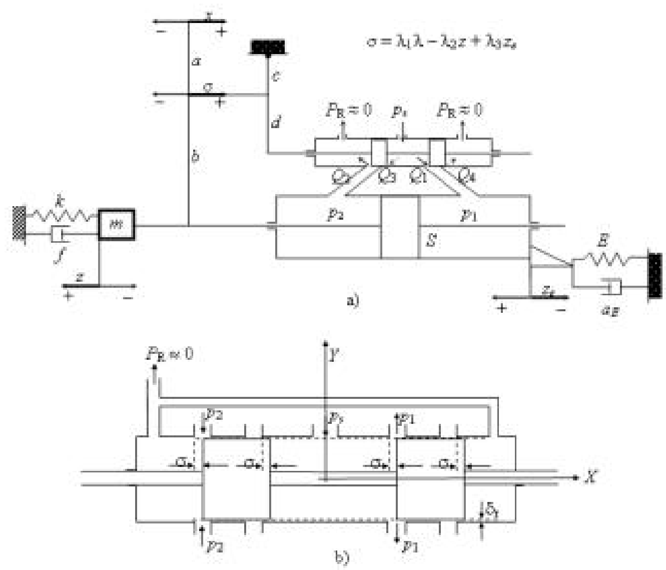

Figure 1: Physical model of a typical MHS: a) spool valve, cylinder with piston, feedback linkage, load, mounting structure; b) ideal "two-land-four-way" spool valve

$$\begin{aligned} c_d w \sigma \sqrt{\frac{2(p_s - p_1)}{\rho}} &= S(\dot{z} - \dot{z}_e) + \frac{V + S(z - z_e)}{B} \dot{p}_1, \\ -c_d w \sigma \sqrt{\frac{2p_2}{\rho}} &= -S(\dot{z} - \dot{z}_e) + \frac{V - S(z - z_e)}{B} \dot{p}_2 \end{aligned} \tag{2.1}$$

and similarly, if $\sigma < 0$

$$\begin{aligned} c_d w\sigma\sqrt{\frac{2}{\rho}p_1} &= S(\dot{z}-\dot{z}_e) + \frac{V+S(z-z_e)}{B}\dot{p}_1, \\ -c_d w\sigma\sqrt{\frac{2(p_s-p_2)}{\rho}} &= -S(\dot{z}-\dot{z}_e) + \frac{V-S(z-z_e)}{B}\dot{p}_2. \end{aligned} \tag{2.1$'$}$$

The parameters are: c_d - volumetric flow coefficient of the valve port; w - valve-port width; p_s - supply pressure; ρ - volumetric density of oil; V - cylinder semivolume; B - bulk modulus of the oil. The variables are: p_1 and p_2 - hydraulic pressures in the chambers of the HC; σ - relative displacement spool-sleeve; z - load displacement; z_e - mounting structure displacement.

The equation of motion of the piston assembly is a force balance equation

$$m\ddot{z} + f\dot{z} + kz = S(p_1 - p_2) \tag{2.2}$$

The dynamics of the feedback deflection z_e is given by an equation involving the stiffness E, an equivalent mass m_e and a damping coefficient a_e

$$m_e\ddot{z}_e + a_e\dot{z}_e + Ez_e = -S(p_1 - p_2). \tag{2.3}$$

An algebraic equation, the feedback linkage equation, completes the differential algebraic mathematical model. It is the equation of the valve opening σ viewed as a linear superposition of three small succesive displacements of input displacement x, output feedback response variable z and feedback deflection z_e induced by structural feedback

$$\sigma = \lambda_1 x - \lambda_2 z + \lambda_3 z_e \tag{2.4}$$

$$\lambda_1 = \frac{b(c+d)}{c(a+b)}, \qquad \lambda_2 = \frac{a(c+d)}{c(a+b)}, \qquad \lambda_3 = \frac{c+d}{c}.$$

The following limitation is to be considered

$$|x| < x_M. \tag{2.5}$$

3. Stability of equilibria

To equations (2.1)-(2.4) corresponds a six-dimensional system of ordinary differential equations that describes the evolution of the servomechanism for $\sigma > 0$. Let

$$C = c_d w \sqrt{\frac{2}{\rho}} \tag{3.1}$$

and introduce the following state variables

$$x_1 = z,\ x_2 = \dot{z},\ x_3 = z_e,\ x_4 = \dot{z}_e,\ x_5 = p_1,\ x_6 = p_2 \tag{3.2}$$

(2.1)-(2.4) give

$$\begin{aligned}
&\dot{x}_1 = x_2\\
&\dot{x}_2 = -\frac{k}{m}x_1 - \frac{f}{m}x_2 + \frac{S}{m}(x_5 - x_6)\\
&\dot{x}_3 = x_4\\
&\dot{x}_4 = -\frac{E}{m_e}x_3 - \frac{a_e}{m_e}x_4 - \frac{S}{m_e}(x_5 - x_6)\\
&\dot{x}_5 = \frac{BS}{V + S(x_1 - x_3)}\left[\frac{C}{S}(\lambda_1 x - \lambda_2 x_1 + \lambda_3 x_3)\sqrt{p_s - x_5} - x_2 + x_4\right]\\
&\dot{x}_6 = \frac{-BS}{V - S(x_1 - x_3)}\left[\frac{C}{S}(\lambda_1 x - \lambda_2 x_1 + \lambda_3 x_3)\sqrt{x_6} - x_2 + x_4\right]
\end{aligned} \tag{3.3}$$

Equilibria $(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5, \hat{x}_6)$ satisfy $\hat{x}_2 = 0$, $\hat{x}_4 = 0$ and

$$\begin{aligned}
&k\hat{x}_1 - S(\hat{x}_5 - \hat{x}_6) = 0\\
&E\hat{x}_3 + S(\hat{x}_5 - \hat{x}_6) = 0\\
&\lambda_1 x - \lambda_2 \hat{x}_1 + \lambda_3 \hat{x}_3 = 0
\end{aligned} \tag{3.4}$$

So

$$\hat{x}_1 = \frac{E\lambda_1 x}{E\lambda_2 + k\lambda_3},\ \hat{x}_3 = -\frac{k}{E}\hat{x}_1,\ \hat{x}_2 = \hat{x}_4 = 0,\ \hat{x}_5 = \hat{x}_6 + \frac{k}{S}\frac{E\lambda_1 x}{E\lambda_2 + k\lambda_3} \tag{3.5}$$

where $\hat{x}_5, \hat{x}_6 \in (0, p_s)$.

Translate such an equilibrium point $(\hat{x}_1, 0, \hat{x}_3, 0, \hat{x}_5, \hat{x}_6)$ given by (3.5) to origin through

$$y_1 = x_1 - \hat{x}_1,\ y_2 = x_2,\ y_3 = x_3 - \hat{x}_3,\ y_4 = x_4,\ y_5 = x_5 - \hat{x}_5,\ y_6 = x_6 - \hat{x}_6 \tag{3.6}$$

and introduce also

$$\hat{x}_0 = \hat{x}_1 - \hat{x}_3 \tag{3.7}$$

The new system will be

$$
\begin{aligned}
&\dot{y}_1 = y_2 := f_1(y_1, \ldots, y_6) \\
&\dot{y}_2 = -\frac{k}{m} y_1 - \frac{f}{m} y_2 + \frac{S}{m}(y_5 - y_6) := f_2(y_1, \ldots, y_6) \\
&\dot{y}_3 = y_4 := f_3(y_1, \ldots, y_6) \\
&\dot{y}_4 = -\frac{E}{m_e} y_3 - \frac{a_e}{m_e} y_4 - \frac{S}{m_e}(y_5 - y_6) := f_4(y_1, \ldots, y_6) \\
&\dot{y}_5 = \frac{BS}{V + S(y_1 - y_3 + \hat{x}_0)} \times \\
&\left[-\frac{C}{S}(\lambda_2 y_1 - \lambda_3 y_3)\sqrt{p_s - y_5 - \hat{x}_5} - y_2 + y_4 \right] := f_5(y_1, \ldots, y_6) \\
&\dot{y}_6 = \frac{BS}{V - S(y_1 - y_3 + \hat{x}_0)} \times \\
&\left[\frac{C}{S}(\lambda_2 y_2 - \lambda_3 y_3)\sqrt{p_6 + \hat{x}_6} + y_2 - y_4 \right] := f_6(y_1, \ldots, y_6)
\end{aligned}
\tag{3.8}
$$

To investigate Lyapunov stability of the zero solution in (3.8) the starting point is the computation of the Jacobian matrix in 0, $A = [a_{ij}]$. Observe that

$$
\begin{aligned}
a_{51} = \frac{\partial f_5}{\partial y_1}(0) = -\frac{BC\lambda_2\sqrt{p_s - \hat{x}_5}}{V + Sx_0}; \quad a_{52} = -\frac{BS}{V + Sx_0}; \\
a_{53} = \frac{BC\lambda_3\sqrt{p_s - \hat{x}_5}}{V + Sx_0}, \quad a_{54} = -a_{52}, \quad a_{55} = a_{56} = 0
\end{aligned}
\tag{3.9}
$$

and

$$
\begin{aligned}
a_{61} = \frac{BC\lambda_2\sqrt{\hat{x}_6}}{V - Sx_0}; \quad a_{62} = \frac{BS}{V - Sx_0}; \quad a_{63} = -\frac{BC\lambda_3\sqrt{\hat{x}_6}}{V - Sx_0}; \\
a_{64} = -a_{62}; \; a_{65} = a_{66} = 0
\end{aligned}
\tag{3.10}
$$

It follows that

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\dfrac{k}{m} & -\dfrac{f}{m} & 0 & 0 & \dfrac{S}{m} & -\dfrac{S}{m} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{E}{m_e} & -\dfrac{a_e}{m_e} & -\dfrac{S}{m_e} & \dfrac{S}{m_e} \\ a_{51} & a_{52} & a_{53} & a_{54} & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & 0 & 0 \end{pmatrix} \tag{3.11}$$

The characteristic polynomial of $\mathbf{A}$ is

$$P(\lambda) = \lambda P_1(\lambda) \tag{3.12}$$

The approach in [5] leads to the following result on stability. Its proof is presented in [6].

Theorem 3.1. *If P_1 in (3.12) is a stable polynomial, equilibria given in (3.5) are Lyapunov stable. When initial conditions are close enough to the equilibrium point the asymptotic behaviour of the solutions is given by*

$$\lim_{t\to\infty} x_1(t) = \hat{x}_1,\ \lim_{t\to\infty} x_2(t) = 0,\ \lim_{t\to\infty} x_3(t) = \hat{x}_3,\ \lim_{t\to\infty} x_4(t) = 0,$$

$$\lim_{t\to\infty} x_5(t) = \hat{x}_5 + \frac{\alpha}{1+d_5},\ \lim_{t\to\infty} x_6(t) = \hat{x}_6 + \frac{\alpha}{1+d_5}.$$

(see [6] for the definitions of d_5 and α).

4. Numerical results

Consider the following values of the system's parameters: $m = 30\,kg$ - equivalent inertia load of primary control surface reduced at the hydraulic servo's piston rod; $m_e = 2\,kg$ - equivalent inertia of hydraulic servo body and attached neighbour structure; $k = 10^5 N/m$ - equivalent aeromounting structure elastic force coefficient; $f = 3 \times 10^3 Ns/m$ - equivalent viscous friction force coefficient; $p_s = 2 \times 10^7 N/m^2$ - supply pressure to valve; $p_R \approx 0$ - return pressure; $S = 10^{-3} m^2$ - piston area; $x_M = 0.03m$ - half of piston stroke $[m]$; $V = 3 \times 10^{-5} m^3$ - semivolume of oil under compression in both cylinder chambers $[m^3]$, $V := S \times x_M$; $B = 6 \times 10^8 N/m^2$ - bulk modulus of oil; $w = 8.5 \times 10^{-4} m$ - area gradient of valve $[m^2/m]$, or valve port width;

$c_d = 0.6$ - valve discharge coefficient; $\lambda_1 = \frac{2}{3}$, $\lambda_2 = \frac{2}{3}$, $\lambda_3 = \frac{4}{3}$ - kinetic feedback coefficients; $\rho = 850 kg/m^3$ - oil density; k_{Q_p} = flow-pressure gain $[m^5/Ns]$; k_Q = valve flow gain $[m^2/s]$.

Usual values of structural damping a_e are given in the literature ([3], [7], [8]) as corresponding to 2% of critical damping ratio so $a_e = 0,04\sqrt{Em_e}$.

Introduce $p \in (0,1)$ by

$$\hat{x}_6 = pp_s \tag{4.1}$$

By the Routh-Hurwitz criterion ([9]) a necessary and sufficient condition that the polynomial

$$P_1(\lambda) = \lambda^5 + \sum_{j=1}^{5} a_j(p,x)\lambda^{5-j}$$

in (3.12) be stable is that the principal minors of the Hurwitz matrix be positive. Thus we must verify:

$$\Delta_1 = a_1 > 0 \text{ and this is indeed the case since } a_1 = \frac{f}{m}$$

$$\Delta_2(p,x) = a_1 a_2(p,x) - a_3(p,x) > 0 \tag{4.2}$$

$$\Delta_3(p,x) = a_1 a_2(p,x) a_3(p,x) + a_1 a_5(p,x) - a_3(p,x)^2 - a_1^2 a_4(p,x) > 0 \tag{4.3}$$

$$\begin{aligned}\Delta_4(p,x) = a_4(p,x)\Delta_3(p,x) - a_5(p,x)[a_1 a_2(p,x)^2 + a_5(p,x) - \\ - a_1 a_4(p,x) - a_3(p,x) a_2(p,x)] > 0\end{aligned} \tag{4.4}$$

$$a_5(p,x) > 0 \tag{4.5}$$

(note that $\Delta_5 = a_4/\Delta_4$).

The relations (4.2)-(4.5) give the stability charts. Some of them are presented in Figures 2, 3, 4, 5.

5. Concluding remarks

The conclusion of this paper, based on a thorough analysis of the Lyapunov stability of equilibria in a six-dimensional nonlinear model of a hydraulic servomechanism, is that the elasticity of mounting structure induces a stabilizing structural feedback in the closed-loop system.

The absence of such quantitative evaluations lead to excessive charge of hydraulic servos with controllers to artificially improve system's stability in various approaches based on automatic control theory.

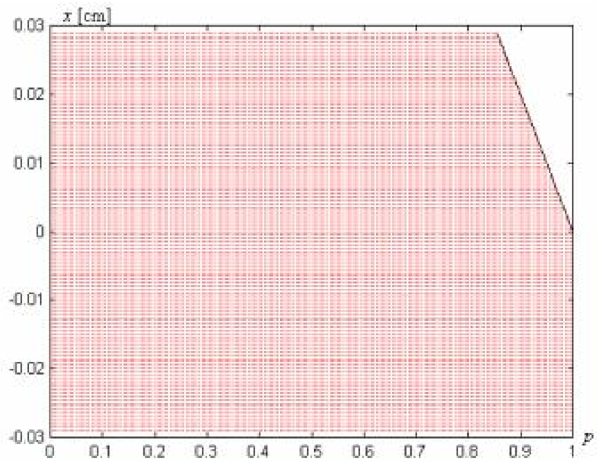

Figure 2: Full parameter space (p, x) stability of the system with nominal mass $m = 30kg\,(E = 55 \times 10^8 N/m,\ \lambda_3 = 0.1)$

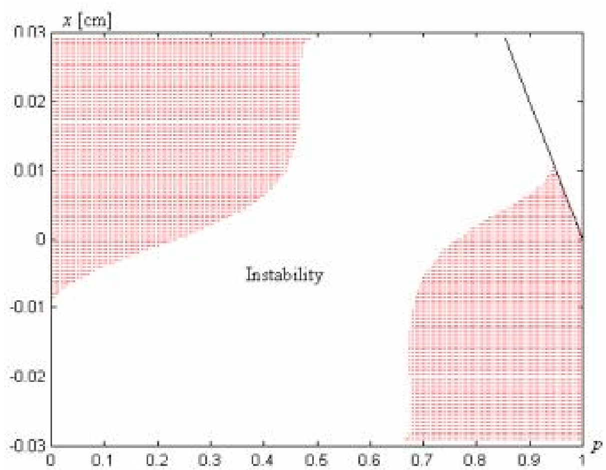

Figure 3: Instability domain for increased mass, $m = 60kg\,(E = 55 \times 10^8 N/m,\ \lambda_3 = 0.1)$

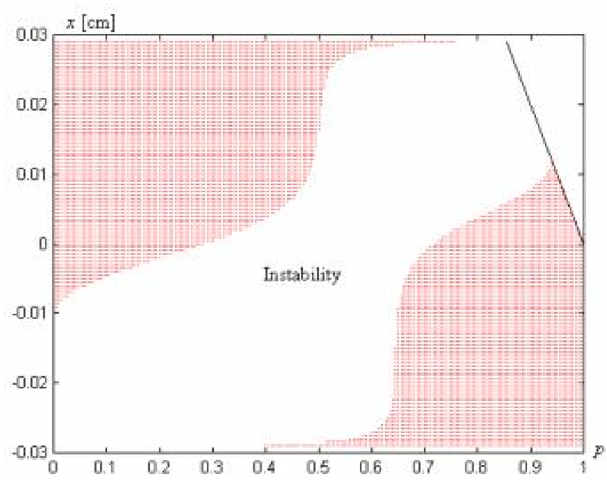

Figure 4: Benefit of increased structural feedback, $\lambda_3 = 4/3$ (the same parameters as in Fig. 3, stability domain is easily increased)

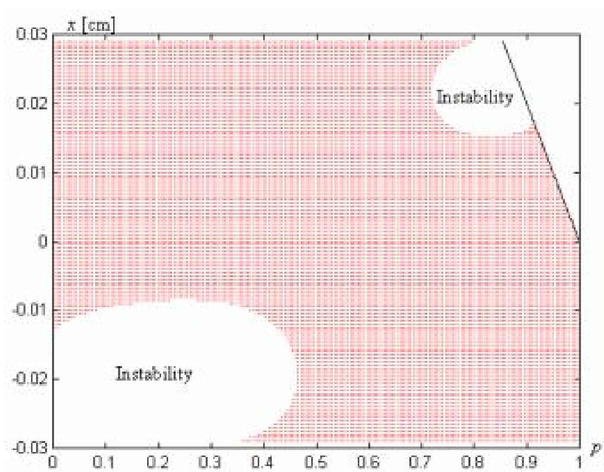

Figure 5: Effect of increasing mounting structure elasticity - E decreased, $E = 55 \times 10^6 N/m$ versus Fig. 3 (the same parameters, $m = 60Kg$, $\lambda_3 = 0.1$)

The analysis performed in the paper took into account two quantities E (the larger is E the lower is the elasticity of the mounting structure) and λ_3, essential for describing servoelastic properties and it was possible to evidence their contribution to stability of equilibria in Figures 2-5.

The model in the present paper is closer to reality than the one in [10] and has also the advantage to take into study internal, natural resources for stabilizing equilibria through adequate design of E and λ_3.

References

[1] R. Taylor, R. W. Pratt and B. D. Caldwell, *Effect of actuator nonlinearities on aeroservoelasticity*, Journal of Guidance, Control, and Dynamics, 19(2), (1996), pp. 309-315.

[2] A. Halanay, C. A. Safta, Stabilization of some nonlinear controlled electrohydraulic servosystems, Applied Mathematics Letters, 18(8), (2005), pp. 911-915.

[3] I. Ursu, M. Vladimirescu, F. Ursu, *About aeroservoelasticity criteria for electrohydraulic servomechanisms synthesis*, Proceedings of 20th Congress of the International Council of the Aeronautical Sciences, ICAS 96, Sorento, Italy, September 8-13, 2, (1996), pp. 2335-2344.

[4] I. Ursu, F. Ursu, *Some absolute stability restrictions in output feedback synthesis of electrohydraulic servos*, Proceedings of the International Conference on Recent Advances in Aerospace Actuation Systems and Components, Toulouse, France, November 24-26, (2004), pp. 173-182.

[5] I. G. Malkin, *Theory of Stability of Motion*, Moscow, Nauka, (in Russian), (English transl. Atomic Energy Comm, Translation, AEC-TR-3352), (1966), pp. 530.

[6] A. Halanay, C. A. Safta, F. Ursu, I. Ursu, Paper in preparation for publication.

[7] F. Kubica, T. Livet, *Design of flight control system for an aeroelastic aircraft*, Proceedings of the 3^{rd} IEEE Control Applications Conference, Glasgow, (1994), pp. 335-340.

[8] Y. Ribakov, J. Gluck, A. M. Reinhorn, *Active viscous damping system for control of MOOF structure*, Earthquake engineering and structural mounting structures, 30, (2001), pp. 195-212.

[9] L. Brand, Differential and Difference Equations, New York, Academic Press, pp. 697, 1966.

Authors:

Ioan Ursu
"Elie Carafoli" National Institute for Aerospace Research
Blvd. Iuliu Maniu 220, Bucharest 061126, ROMANIA
email:*iursu@aero.incas.ro*

Felicia Ursu
"Elie Carafoli" National Institute for Aerospace Research
Blvd. Iuliu Maniu 220, Bucharest 061126, ROMANIA
email:*fursu@aero.incas.ro*

Andrei Halanay
Department of Mathematics 1, University Politehnica of Bucharest
Splaiul Independenţei 313, Bucharest, 060042, ROMANIA
email:*halanay@mathem.pub.ro*

Carmen Anca Safta
Department of Hydraulic and Hydraulic Machinery, University Politehnica of Bucharest
Splaiul Independenţei 313, Bucharest, 060042, ROMANIA
email:*saphta@hydrop.pub.ro*