

## A BOUNDARY ELEMENT APPROACH FOR THE COMPRESSIBLE FLUID FLOW AROUND OBSTACLES

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**ABSTRACT.** The paper presents an application of the boundary element method to solve the problem of the compressible fluid flow around obstacles, so to find the perturbation produced by the obstacles and the fluid action on them. The mathematical model of the problem is equivalent with a boundary singular integral equation which is solved using different kinds of boundary elements for smooth profiles and for profiles with cusped trailing edge. Based on the methods exposed some computer codes in MATHCAD are developed. For some particular cases there exist exact solutions and so with the computer codes we make a comparison between the numerical solutions obtained and the exact ones. The comparison study shows a good agreement between them. The paper is also focused on the study of the influence of the compressibility on this motion. Using the computer codes in the case of profiles with cusped trailing edge there are obtained numerical solutions for the local pressure coefficient and for the lifting force for different Mach numbers. These results are used to study the influence of the compressibility on the fluid action over the obstacle.

*2000 Mathematics Subject Classification:* 74S15.

### 1. INTRODUCTION

The boundary integral method (BEM) is a modern theory of great efficiency used to solve boundary value problems for systems of partial differential equations. The BEM has a real advantage over other numerical methods because it reduces the problem dimension by one. To achieve this reduction of dimension it is necessary to obtain, using an indirect or a direct technique,

an equivalent model of the problem, in terms of boundary integral equations ( see [1], [2]).

In this paper we apply this method for solving the two-dimensional problem of the compressible fluid flow around an obstacle. We consider that the uniform, steady, potential motion of an ideal inviscid fluid of subsonic velocity  $\bar{U}_\infty \bar{i}$ , pressure  $p_\infty$  and density  $\rho_\infty$  is perturbed by the presence of a fixed body of a known boundary, assumed to be smooth and closed. We want to find out the perturbed motion, and the fluid action on the body.

Because the body is 2-dimensional, the boundary elements used to solve the singular boundary integral the problem is reduced at are unidimensional, and have the nodes on the external surface of the body. In the herein paper we use constant and linear isoparametric boundary elements to solve the problem. We so reduce by discretization the integral equation to an algebraic system and the solutions of this system are then used to calculate the perturbation velocity and the pressure coefficient over the body.

Using dimensionless variables we have for the components of the perturbation velocity( $U, V$ ) the equations:

$$\begin{aligned} \beta^2 \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\ \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} &= 0 \end{aligned} \quad (1)$$

where  $\beta = \sqrt{1 - M^2}$ ,  $M$  being Mach number, and the boundary condition:

$$(1 + U) N_x + N_y = 0 \quad (2)$$

where  $N_x, N_y$  are the components of the normal vector outward the fluid (inward the body).

Using the following change of coordinates:  $x = X$ ,  $y = \beta Y$ ,  $u = \beta U$ ,  $v = V$ , the system (1) becomes:

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} &= 0 \end{aligned} \quad (3)$$

and the boundary condition:

$$(\beta + u) n_x + \beta^2 v n_y = 0 \quad \text{on } C \quad (4)$$

It is also required that the perturbation velocity vanishes at infinity:  $\lim_{\infty} (u, v) = 0$ .

Using a direct integral method a boundary singular integral equation, formulated in velocity vector terms, is obtained in [6] for the considered problem. First  $u(\bar{x}_0)$  and  $v(\bar{x}_0)$ , the components of the perturbation velocity in any point of the boundary,  $\bar{x}_0 \in C$ , are deduced, and then, denoting  $G = (\beta + u)n_y - vn_x$ , the following representations are obtained:

$$\begin{aligned} u(\bar{x}_0) &= \beta + 2 \int_C^/ \left( v^* - M^2 \frac{n_x n_y}{n_x^2 + \beta^2 n_y^2} u^* \right) G ds \\ v(\bar{x}_0) &= 2 \int_C^/ \left( -u^* - M^2 \frac{n_x n_y}{n_x^2 + \beta^2 n_y^2} v^* \right) G ds \end{aligned} \quad (5)$$

and the singular integral equation:

$$\frac{1}{2} G(\bar{x}_0) + \int_C^/ (u^* n_x^0 + v^* n_y^0) G ds + \int_C^/ M^2 \frac{n_x n_y}{n_x^2 + \beta^2 n_y^2} (v^* n_x^0 - u^* n_y^0) G ds = \beta n_y^0 \quad (6)$$

where  $u^*(\bar{x}, \bar{x}_0) = \frac{1}{2\pi} \frac{x-x_0}{(x-x_0)^2 + (y-y_0)^2}$ ,  $v^*(\bar{x}, \bar{x}_0) = \frac{1}{2\pi} \frac{y-y_0}{(x-x_0)^2 + (y-y_0)^2}$ , are the fundamental solutions (see[5]), and  $\bar{n}_0 = \bar{n}(\bar{x}_0) = n_x^0 \bar{i} + n_y^0 \bar{j}$ .

The sign “ / ” denotes the principal value in the Cauchy sense of an integral,  $M$  is the Mach number for the unperturbed uniform motion, and  $\beta$  has the well known expression for subsonic flow,  $\beta = \sqrt{1 - M^2}$ . For simplifying the writing we shall not use the prim sign to specify that an integral must be understand in its Cauchy sense.

Equation (6) is the singular integral equation of the problem (for more information about obtaining this singular integral equation see [6] ). For  $M = 0$  we get the singular equation for the incompressible case.

When the obstacle has a cusped trailing edge (like the Jukovsky profile), we have the following expression for the dimensionless circulation:  $\Gamma = -\frac{1}{\beta} \int_C G ds$ .

In order to solve the integral equation we consider two cases: first we use constant boundary elements, so we approximate the boundary by a polygonal line and we consider that the unknown has a constant value on each of the

resulting segments, and then we use linear isoparametric boundary elements to solve the singular boundary integral equation.

## 2. CASE OF CONSTANT BOUNDARY ELEMENTS

In the boundary element approach used herein, for solving the integral equation (6), the boundary  $C$  is divided into  $N$  linear segments, noted  $L_j, j = \overline{1, N}$ , with ends in  $\bar{x}^j, \bar{x}^{j+1}, j = \overline{1, N}, \bar{x}^{N+1} = \bar{x}^1$ ; the extremes of the panels being situated on  $C$ . We then consider that the unknown function  $G$  is constant on each  $L_j$ , and equal with the value taken in the midpoint of the segment, noted  $\bar{x}_j^0 = \frac{\bar{x}^j + \bar{x}^{j+1}}{2}, j = \overline{1, N}$ .

In (6) we consider than  $\bar{x}^0 = \bar{x}_i^0, i = \overline{1, N}$  and we obtain the linear algebraic system:

$$\frac{1}{2}G_i + \sum_{j=1}^N A_{ij}G_j = B_i \quad (7)$$

where  $G_i = G(\bar{x}_i^0)$ . The coefficients may be computed (see [7]) with the following formulas:

$$B_i = \beta n_y(\bar{x}_i^0) \quad (8)$$

$$A_{ij} = -n_x(\bar{x}_i^0)U_{ij} - n_y(\bar{x}_i^0)V_{ij} - M^2 \frac{n_x(\bar{x}_j^0)n_y(\bar{x}_j^0)}{n_x^2(\bar{x}_j^0) + \beta^2 n_y^2(\bar{x}_j^0)} \cdot [n_x(\bar{x}_i^0)V_{ij} - n_y(\bar{x}_i^0)U_{ij}] \quad (9)$$

where  $U_{ij}, V_{ij}$  depend of the fundamental solutions and have the expressions:

$$U_{ij} = \int_{L_j} u^*(\bar{x}, \bar{x}_i^0) ds, \quad V_{ij} = \int_{L_j} v^*(\bar{x}, \bar{x}_i^0) ds \quad (10)$$

These integrals can be calculated numerically with a computer but they can also be calculated analytically. We prefer the last possibility because it is an exact method for evaluating integrals.

Denoting

$$4a_j = l_j^2, \quad b_{ij} = (x_j^0 - x_i^0)(x_2^j - x_1^j) + (y_j^0 - y_i^0)(y_2^j - y_1^j),$$

$$c_{ij} = (x_j^0 - x_i^0)^2 + (y_j^0 - y_i^0)^2$$

we get:

$$\begin{aligned} U_{ij} &= \frac{l_j}{4\pi} \left[ (x_j^0 - x_i^0) I_0^{ij} + \frac{x_2^j - x_1^j}{2} I_1^{ij} \right] \\ V_{ij} &= \frac{l_j}{4\pi} \left[ (y_j^0 - y_i^0) I_0^{ij} + \frac{y_2^j - y_1^j}{2} I_1^{ij} \right] \end{aligned} \quad (11)$$

where

$$I_k^{ij} = \int_{-1}^1 \frac{t^k dt}{a_j t^2 + b_{ij} t + c_{ij}}, \quad k = \overline{0, 1} \quad (12)$$

For  $i \neq j$  these last integrals are usual integrals and after calculating them we have:  $I_0^{ij} = \frac{2}{\sqrt{-\Delta_{ij}}} \operatorname{arctg} \frac{\sqrt{-\Delta_{ij}}}{c_{ij} - a_j}$ ,  $I_1^{ij} = \frac{1}{2a_j} \ln \frac{a_j + b_{ij} + c_{ij}}{a_j - b_{ij} + c_{ij}} - \frac{b_{ij}}{a_j \sqrt{-\Delta_{ij}}} \operatorname{arctg} \frac{\sqrt{-\Delta_{ij}}}{c_{ij} - a_j}$ , where  $\Delta_{ij} = b_{ij}^2 - 4a_j c_{ij}$ . For  $i = j$  we have singular integrals. After evaluating their principal values in the Cauchy sense we get:  $I_0^{jj} = -\frac{2}{a_j}$ ,  $I_1^{jj} = 0$ .

So all the coefficients in system (7) can be evaluated using the coordinates of the nodes chosen for the boundary discretization.

After solving the system we can evaluate the perturbing velocity in the nodes on the boundary by means of the following relations :

$$\begin{aligned} \beta + u_i &= \frac{\beta^2 n_y(\bar{x}_i^0) G_i}{n_x^2(\bar{x}_i^0) + \beta^2 n_y^2(\bar{x}_i^0)} \\ v_i &= \frac{n_x(\bar{x}_i^0) G_i}{n_x^2(\bar{x}_i^0) + \beta^2 n_y^2(\bar{x}_i^0)} \end{aligned} \quad (13)$$

The components of the perturbation velocity are then used to calculate the local pressure coefficient.

### 3. TESTING THE METHOD - CASE OF A NONLIFTING BODY

We test the method solving this algebraic system in a particular case, for a circular profile and an incompressible fluid, case in which we know an exact solution. Considering different values for the Mach number we can study the influence of the compressibility over this motion. For the case of the circular obstacle to incompressible flow an exact solution exists (see [5]).

As we know, the pressure coefficient is given by:  $C_p = \frac{p_1 - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = 1 - \frac{V^2}{U_\infty^2}$ , where  $V$  is the global velocity:  $V = U_\infty (\bar{i} + \bar{v})$ .

So, after the components of the velocity are found, the pressure coefficient can be calculate with the formula:

$$C_p(\bar{x}_i) = - (u(\bar{x}_i)^2 + v(\bar{x}_i)^2) - 2u(\bar{x}_i). \quad (14)$$

The exact solution furnishes the following expressions for the components of the velocity in a point  $M(R, \theta)$  situated on the circle  $C(O, R)$  (the quantities we use are also dimensionless):  $u = -\cos 2\theta$ ,  $v = -\sin 2\theta$ . For the pressure coefficient we have:

$$C_p = -1 + 2 \cos 2\theta \quad (15)$$

With two computer codes, one for the exact solution and other for the method expose I found the nodal values for the case of a circular obstacle. The results are compared through the pressure coefficient. Comparisons between the exact values of the pressure coefficient (using (14)) and the values calculated by the means of the boundary element method, obtained for 20 nodes on the boundary, are performed in Fig.1. Comparing the values obtained for the exact solution and the calculated one we remark a high degree of accuracy.

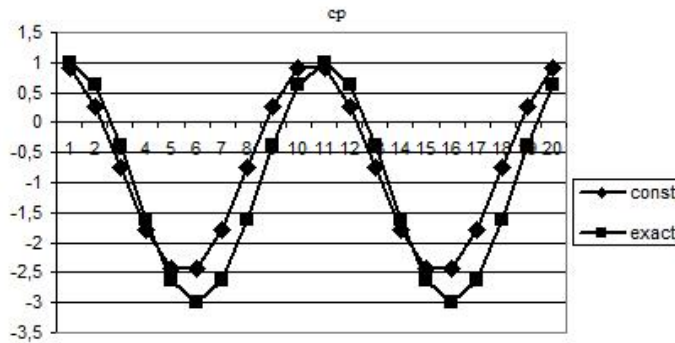


Figure 1: The local pressure coefficient for the circular obstacle

#### 4. TESTING THE METHOD - CASE OF A LIFTING BODY

For the case of the fluid flow in the presence of the Jukovsky profile the problem has not a unique solution, so we have to introduce a Kutta-Jukovsky condition: the pressure at  $P_s$ , situated on the upper surface, adjacent to the trailing edge, noted  $P_f$ , must be equal with the pressure at  $P_i$ , situated on the lower surface, when  $P_s, P_i \longrightarrow P_f$ .

We deduce a condition, in terms of the unknown  $G$ :

$$G(P_s) + G(P_i) = 0 \quad (16)$$

So, for obtaining the unique solution of the problem in the case of a Jukovsky profile, we have to complete the algebraic system (7), with the condition above, and to introduce a new variable,  $\lambda$  -a regularization variable. We finally get the system:

$$\begin{aligned} \lambda + \frac{1}{2}G_i + \sum_{j=1}^N A_{ij}G_j &= B_i, \quad i = \overline{1, N} \\ G_1 + G_N &= 0 \end{aligned} \quad (17)$$

for the unknowns  $G_1, G_2, \dots, G_N, \lambda$ .

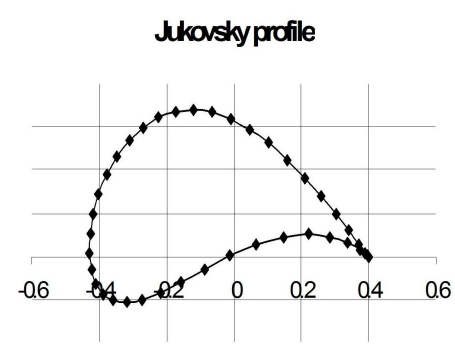
Solving this system we can observe that so as  $N$  grows the value for  $\lambda$  goes to zero. After the determination of the unknown  $G_i$  the circulation is obtained with the formula:

$$-\beta\Gamma = \sum_{j=1}^N l_j G_j, \quad (18)$$

where  $l_j$  represents the length of the segment  $L_j$ , and the velocity with the same relations (12) as in the above paragraph.

In order to test the method we shall consider the motion in the presence of a Jukovsky profile. In this case and for the inviscid fluid, it is known the exact solution of the problem, which can be found in [5], [8]. In [8] it is proposed a program in MATLAB which calculate the exact values for the pressure coefficient for the next Jukovsky profile, for which the complex coordinate of a point on the profile is:

$$z(\theta) = \tilde{z}(\theta) + \frac{b^2}{\tilde{z}(\theta)} \quad (19)$$



where  $\tilde{z}(\theta) = \cos\theta + i\sin\theta + x_0 + iy_0$ ,  $b = 0.8$ ,  $y_0 = 0.189$ ,  $x_0 = b - \sqrt{1 - y_0^2}$ . The profile given by (18) is represented in the figure bellow

Using MATHCAD we made a program that calculates the coefficients of the system, the unknowns, the components of the velocity and the pressure coefficient at the nodal points, program based on the method proposed in this section. There were used 40 points for the discretization of the boundary, and constant boundary elements. With the program made we can also calculate the circulation and the lift.

We test the method comparing the exact solution for the pressure coefficient with the numerical one in the case of the Jukovsky profile given by (18) .

From Fig.2, made for this case, we can observe that the calculated and the analytical values of the pressure coefficient are very close, except for the trailing edge, facts that shows the great accuracy of the method proposed. From the same figure we can also see the difference between the two points of minimum, on the upper surface and the lower one, so the appearance of the lifting force. It appears reasonable to expect better results in the vicinity of the trailing edge by using more control points in this area or other kinds of boundary elements such as the linear or the curved ones.

The problem of the 2D potential flow around Jukovsky profiles (with cusped trailing edge) was also studied in many papers, for example in [3], but the fluid was considered incompressible, the unknown function was the stream function, and the pressure coefficient could not be calculated in the trailing edge. In the present paper we can notice two major differences: there are used primary functions (the components of the velocity) as unknowns of the problem, and the fluid is compressible (the most important).



With the aid of the program based on the method proposed in the present paper we can also study the influence of the compressibility on the pressure coefficient, and on the lifting force that appears, by considering different values for the Mach number. Considering the effect of the compressibility we have for the pressure coefficient the formula (see [6] ):

$$C_p = \frac{2}{\gamma M^2} \left\{ -1 + \frac{\gamma - 1}{2} M^2 \left( 1 - \frac{v^2}{U_\infty^2} \right)^{\frac{\gamma}{\gamma-1}} \right\} - 1 \quad (20)$$

For the case of a barotrop fluid and the coefficient ( for air) we get the results in the Fig. 3 and Fig.4. We can observe two important things: the difference between the minimum values grows as Mach number grows either, and the motion loses its stability as Mach number goes to its critical value (as in [4]). In Fig. 5 there is represented the difference between the minimum values for different Mach numbers: (col.1-for M=0; col.2-for M=0.2; col.3-for M=0.4;col.4- for M=0.5).

In Figure 6 and Figure 7 there are represented the circulation, calculated with (17), and the lift for different Mach numbers. We remark that both grow as Mach number grows either.

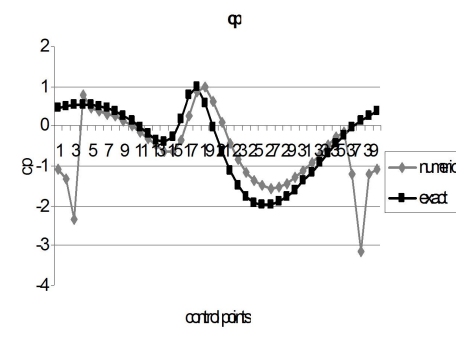


Figure 2: The pressure coefficient for the control points ( $M=0$ )

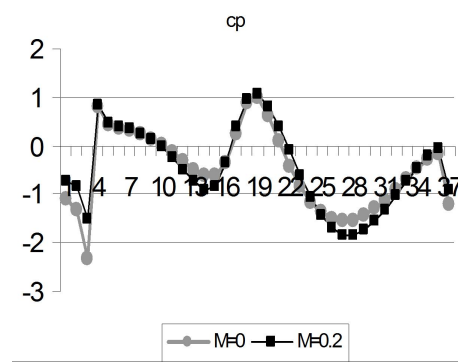


Figure 3: The pressure coefficient for the control points ( $M=0$ ,  $M=0.2$ )

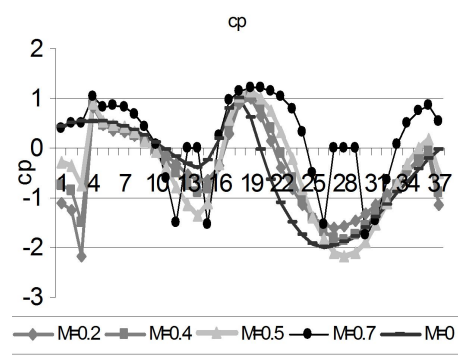


Figure 4: The pressure coefficient for the control points (The pressure coefficient for different Mach numbers).

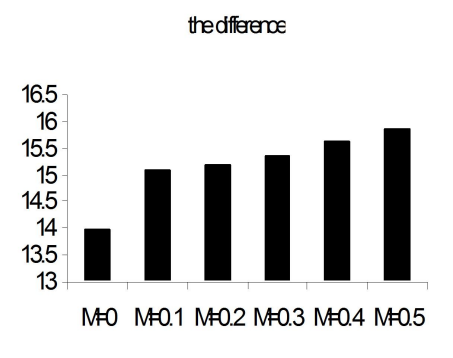


Figure 5: The pressure coefficient for the control points (The values of the difference between the minimum values of the pressure coefficient).

Figure 6: The circulation for different values of Mach number  $M$ .

Figure 7: The lift for different values of Mach number  $M$ .



For evaluating the integrals we use a local system of coordinates (see [1], [2]) which has the origin in the first node of an element, and so we have, for a boundary element, the relations:  $x = x_j^1\varphi_1 + x_j^2\varphi_2$ ,  $y = y_j^1\varphi_1 + y_j^2\varphi_2$ , where  $\varphi_1, \varphi_2$  are the form functions given by  $\varphi_1 = 1-t, \varphi_2 = t, t \in [0, 1]$ .

Using isoparametric boundary elements (see[1] ) we have, for the unknown  $G$ , the local representation:  $G = G_j^1\varphi_1 + G_j^2\varphi_2$ , where  $G_j^1, G_j^2$  are the nodal values of the unknown, it means the values of  $G$  at the extremes of the boundary element  $L_j$ , in the local numbering. These values satisfy the relations:  $G_j^2 = G_{j+1}^1, \forall i \in \{1, 2, \dots, N-1\}$ , and  $G_N^2 = G_1^1$ , for the case of a smooth boundary so a nonlifting body

Using the linear interpolation for  $G$ , and using the local system of coordinates the system (20) becomes:

$$\begin{aligned}
 & \frac{1}{2}G(\bar{x}_i) + \sum_{j=1}^N G_j^1 l_j \left[ n_x^i \int_0^1 u^* \varphi_1 dt + n_y^i \int_0^1 v^* \varphi_1 dt \right] + \\
 & + \sum_{j=1}^N G_j^1 l_j \left[ M^2 \frac{n_x^j n_y^j}{(n_x^j)^2 + \beta^2 (n_y^j)^2} \left( n_x^i \int_0^1 v^* \varphi_1 dt - n_y^i \int_0^1 u^* \varphi_1 dt \right) \right] + \\
 & + \sum_{j=1}^N G_j^2 l_j \left[ n_x^i \int_0^1 u^* \varphi_2 dt + n_y^i \int_0^1 v^* \varphi_2 dt \right] + \\
 & + \sum_{j=1}^N G_j^2 l_j \left[ M^2 \frac{n_x^j n_y^j}{(n_x^j)^2 + \beta^2 (n_y^j)^2} \left( n_x^i \int_0^1 v^* \varphi_2 dt - n_y^i \int_0^1 u^* \varphi_2 dt \right) \right] = \beta n_y^i
 \end{aligned} \tag{22}$$

where  $i = \overline{1, N}, l_j = \left( (x_j^2 - x_j^1)^2 + (y_j^2 - y_j^1)^2 \right)^{\frac{1}{2}} = \|\bar{x}_j^2 - \bar{x}_j^1\|$ .

With the notations:

$$\begin{aligned}
 a_j &= l_j^2, b_{ij} = (x_j^1 - x_i) (x_j^2 - x_j^1) + (y_j^1 - y_i) (y_j^2 - y_j^1), \\
 c_{ij} &= (x_j^1 - x_i)^2 + (y_j^1 - y_i)^2 \\
 I_k^{ij} &= \int_0^1 \frac{t^k dt}{a_j t^2 + b_{ij} t + c_{ij}}, \quad k = \overline{0, 2} \\
 r_{ij} &= \frac{l_j}{2\pi} [(x_j^2 - x_j^1) (I_1^{ij} - I_2^{ij}) + (x_j^1 - x_i) (I_0^{ij} - I_1^{ij})] \\
 rr_{ij} &= \frac{l_j}{2\pi} [(x_j^2 - x_j^1) I_2^{ij} + (x_j^1 - x_i) I_1^{ij}]
 \end{aligned} \tag{23}$$

$$\begin{aligned}
 s_{ij} &= \frac{l_j}{2\pi} [(y_j^2 - y_j^1) (I_1^{ij} - I_2^{ij}) + (y_j^1 - y_i) (I_0^{ij} - I_1^{ij})] \\
 ss_{ij} &= \frac{l_j}{2\pi} [(y_j^2 - y_j^1) I_2^{ij} + (y_j^1 - y_i) I_1^{ij}] \\
 A_{ij} &= \left( n_x^i - M^2 n_y^i \frac{n_x^j n_y^j}{(n_x^j)^2 + \beta^2 (n_y^j)^2} \right) r_{ij} + \left( n_y^i + M^2 n_x^i \frac{n_x^j n_y^j}{(n_x^j)^2 + \beta^2 (n_y^j)^2} \right) rr_{ij} \\
 B_{ij} &= \left( n_x^i - M^2 n_y^i \frac{n_x^j n_y^j}{(n_x^j)^2 + \beta^2 (n_y^j)^2} \right) s_{ij} + \left( n_y^i + M^2 n_x^i \frac{n_x^j n_y^j}{(n_x^j)^2 + \beta^2 (n_y^j)^2} \right) ss_{ij}
 \end{aligned} \tag{24}$$

we get the following equivalent form for system(20 ):

$$\frac{1}{2} G_i + \sum_{j=1}^N A_{ij} G_j^1 + \sum_{j=1}^N B_{ij} G_j^2 = \beta n_y^j \tag{25}$$

## 5.2. NUMERICAL EVALUATION OF THE COEFFICIENTS

As in the first case, the integrals that appear can be exactly evaluated for the case when they are not singular, in fact for the case when the node  $i$  is not one of  $L_j$  extremes.

In the global system of numbering the extremes of  $L_j$  are noted  $\bar{x}^j, \bar{x}^{j+1}, j = \overline{1, N}$ , with  $\bar{x}^{N+1} = \bar{x}^1$ . So using only the global system of numbering we have the following relations:  $\bar{x}_j^1 = \bar{x}^j, \bar{x}_j^2 = \bar{x}^{j+1}$ .

The analytical expressions for these integrals are, when  $i \neq j$  and  $i \neq j+1$ :

$$\begin{aligned}
 I_0^{ij} &= \frac{1}{\sqrt{a_j c_{ij} - b_{ij}^2}} \arctan \frac{\sqrt{a_j c_{ij} - b_{ij}^2}}{c_{ij} + b_{ij}}, \\
 I_1^{ij} &= \frac{1}{2a_j} \ln \frac{a_j + 2b_{ij} + c_{ij}}{c_{ij}} - \frac{b_{ij}}{a_j \sqrt{a_j c_{ij} - b_{ij}^2}} \arctan \frac{\sqrt{a_j c_{ij} - b_{ij}^2}}{c_{ij} + b_{ij}}, \\
 I_2^{ij} &= \frac{1}{a_j} - \frac{b_{ij}}{a_j^2} \ln \frac{a_j + 2b_{ij} + c_{ij}}{c_{ij}} + \frac{2b_{ij}^2 - a_j c_{ij}}{a_j^2} I_0^{ij}. \tag{26}
 \end{aligned}$$

Taking into account the going sense on C we have the following relations for the components of the normal in  $\bar{x}_j$  :  $n_x^j = \frac{y_j^2 - y_i^1}{l_j}$ ,  $n_y^j = \frac{x_j^1 - x_i^2}{l_j}$ ,  $\forall j = \overline{1, N}$ .

So all the coefficients resulting from the nonsingular integrals can be computed analytically.

For the case of singular integrals Considering now that  $\bar{x}_i \in L_j$  we calculate the singular integrals occurring in (20). Using the same notations as before, but calculating the Cauchy principal values for the integrals  $I_k^{ij}$ , we find for  $i = j$  and for  $i = j + 1$ ,  $j = \overline{1, N}$ , the following results:  $I_0^{jj} = I_1^{jj} = -\frac{1}{l_j^2}$ ,  $I_2^{jj} = 0$ ,  $I_0^{j+1j} = I_1^{j+1j} = -\frac{1}{l_j^2}$ ,  $I_2^{j+1j} = 0$ .

Returning to the global system of notation, that mean  $G(\bar{x}_i) = G_i$ ,  $i = \overline{1, N}$ , we have  $G_j = G_j^1 = G_{j-1}^2$ ,  $j = \overline{2, N}$ ,  $G_1 = G_1^1 = G_N^2$ , for the case of a nonlifting body; so system (20) has  $N$  equations with  $N$  unknowns-the  $N$  nodal values of  $G$ . All the coefficients that occur in system (20) can be computed analytically and for all these calculus and also for solving the system a computer code can be used.

If we denote

$$C_{ij} = A_{ij} + B_{ij-1}, \quad i = \overline{2, N}, \quad A_{i1} = A_{i1} + B_{iN}, \quad B_i = \beta n_y^i, \quad i = \overline{1, N} \tag{27}$$

we finally obtain the following equivalent form for system (20), in terms of nodal unknowns:

$$\sum_{j=1}^N C_{ij} G_j = B_i, \quad i = \overline{1, N} \tag{28}$$

For the case of a profile with a cusped trailing edge we must take into account a Kutta -Jukovsky condition, for getting a unique solution of the

problem, as in the first case. So we consider that  $G_j = G_j^1 = G_{j-1}^2, j = \overline{2, N}, G_1 = G_1^1, G_{N+1} = G_N^2$  and a new equation: to complete the above system:  $G_1 + G_{N+1} = 0$

After solving the system the problem is reduced at we can evaluate the perturbing velocity in the nodes of the boundary by means of relation (12), for both cases, lifting and nonlifting profiles, and after the components of the velocity are found, the pressure coefficient can be calculate too.

### 5.3. NUMERICAL RESULTS

In order to test the method, we consider the case of the circular obstacle and an incompressible flow. A computer code in MATHCAD evaluate the numerical values for the nodal perturbation components of the velocity and for the nodal values of the local pressure coefficient. For this case the exact solution of the problem, the numerical one obtained with constant boundary elements as in section 3, and the numerical solution obtained for the case of linear boundary elements are compared, through the pressure coefficient, in Fig.8, and as we see they are in very good agreement. As we can see, the calculated and the analytical values of the pressure coefficient are very close, in both cases, but a higher degree of accuracy is obtained by the use of linear elements. We can also observe that a small numbers of elements (20) are sufficient for obtaining satisfactory results. As expected, better results are obtained when using more nodes or curved boundary elements which allow a better approximation of the geometry.

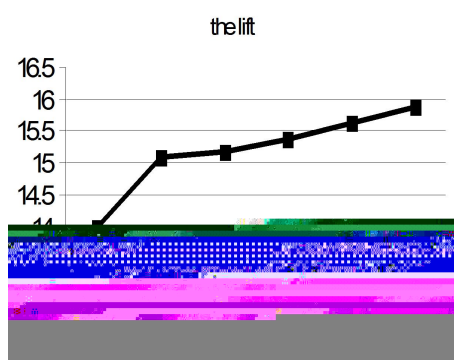


Figure 8: The pressure coefficient obtained using 20 nodes(case of a circular obstacle)



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