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FUZZY HYPOTHESIS TESTING WITH FUZZY DATA BY USING FUZZY P-VALUE

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Abstract. One of the most important problems in statistical inference is testing of statistical hypothesis. Usually, the underlying data and the hypotheses are assumed to be precise. But, in many situations it is much more realistic in general to consider fuzzy concepts. This paper is devoted to the problem of testing hypotheses when the both hypotheses and data are fuzzy. We first extension the notion of fuzzy p-value which is appropriates for this case and then present an approach for this testing by comparing the obtained fuzzy p-value and fuzzy significance level, based on a comparison of two fuzzy sets.

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1. Introduction

In the traditional approach for hypotheses testing, all of the concepts are assumed to be precise and well-defined. These assumptions, sometimes, force a statistician to make decision procedure in an unrealistic manner, because in realistic problems we may come across fuzzy data and fuzzy hypotheses. The problem of statistical hypotheses testing when the hypotheses and data are fuzzy has been studied by some authors. Arnold (1996) studied on fuzzy hypotheses testing with crisp data. The problem of testing fuzzy hypotheses when the observations are crisp were considered by Taheri and Behboodian (1999). Torabi et al. (2006) used Neyman-Pearson Lemma for fuzzy hypotheses testing with vague data. Casals et al. (1986) considered the problem of testing hypotheses when the available data are fuzzy and extended both Neyman-Pearson and Bayes theories to this framework. Casals (1993) also studied on the same problem in the context of fuzzy decision problems. Arefi and Taheri (2011) studied testing fuzzy hypotheses using fuzzy data based on fuzzy test statistic.

For the first time, Filzmoser and Viertl (2004) used fuzzy p-value in the testing of hypotheses when the observations are fuzzy and hypotheses are crisp. Denoeux et al. (2005) proposed the fuzzy p-value for nonparametric tests by a rank-based statistic approach with fuzzy data. Parchami et al. (2010) considered p-value in testing of fuzzy hypotheses when data are crisp. They also presented an approach for this testing by comparing the obtained p-value and a fuzzy significance level. None of the above authors have considered the fuzzy p-value when both the hypotheses and data are fuzzy, by now. In this paper, we introduce the fuzzy p-value such that it is useful for testing of hypotheses when the hypotheses and data are both fuzzy.

The remainder of this paper is organized as follows. In Section 2 we present some preliminaries. Section 3 is concerned with classical hypotheses testing. Section 4 gives some concepts of fuzzy hypotheses testing problems. In section 5, we introduce the fuzzy p-value for testing of fuzzy hypotheses when data are fuzzy. Numerical examples to show the performance of the method are also proposed. A brief conclusion is given in Section 6.

2. Preliminaries

Let Ω be an universal set and $F(\Omega) = \{\mu |\ \mu : \Omega \longrightarrow [0,1]\}$. Any $\mu \in F(\Omega)$ is said to be a fuzzy set on Ω. In particular, let $\mathbb{R}$ be the set of real numbers. For convenient, we will use the following notations (compare to Parchami et al., 2010):

$$F_C(\mathbb{R}) = \{\mu |\ \mu : \mathbb{R} \longrightarrow [0,1],\quad \mu\ is\ a\ continuous\ function\},$$

$$F_S(\mathbb{R}) = \{S(a,b) |\ a, b \in \mathbb{R},\ a \leq b\},$$

$$F_B(\mathbb{R}) = \{B(a,b) |\ a, b \in \mathbb{R},\ a \leq b\},$$

$$F_T(\mathbb{R}) = \{T(a,b,c) |\ a, b, c \in \mathbb{R},\ a \leq b \leq c\},$$

where

$$S(a,b)(x) = \begin{cases} 1 & if\ x \leq a, \\ \frac{(x-b)}{(a-b)} & if\ a < x \leq b, \\ 0 & if\ x > b, \end{cases}$$

$$B(a,b)(x) = \begin{cases} 0 & if\ x < a, \\ \frac{(x-a)}{(b-a)} & if\ a < x \leq b, \\ 1 & if\ x \geq b. \end{cases}$$

and

$$T(a,b,c)(x) = \begin{cases} \frac{(x-a)}{(b-a)} & if\ a < x \leq b, \\ \frac{(x-c)}{(b-c)} & if\ b < x \leq c, \\ 0 & elsewhere. \end{cases}$$

$T(a,a,a)$ denotes the indicator function of a, i.e. $I_{\{a\}}$.

Definition 1. (Zimmermann, 1984). If $\mu \in F_C(\mathbb{R})$ then,

(a) μ is called normal, if there exists $x \in \mathbb{R}$ such that $\mu(x) = 1$;

(b) μ is called convex, if

$$\mu(\lambda x + (1-\lambda)y) \geq \min(\mu(x), \mu(y)), \quad \forall\ x, y \in \mathbb{R}, \quad \forall\ \lambda \in [0,1];$$

(c) The support of μ is the crisp set given by $Supp(\mu) = \{x|\mu(x) > 0\}$;

(d) The δ-cut of μ is the crisp set given by

$$C_\delta(\mu(\cdot)) = [C_\delta^L(\mu(\cdot)), C_\delta^U(\mu(\cdot))] := \{x|\mu(x) \geq \delta\}, \quad for\ all\ \delta \in (0,1].$$

Definition 2. (Zimmermann, 1984). A non-precise number x^* is a fuzzy subset of $\mathbb{R}$ whose membership function $\mu(.)$ obeys the following conditions:

(1) $\forall\ \delta \in (0,1]$, the δ-cut $C_\delta(\mu(\cdot))$ is a finite unioin of compact intervals $[a_{\delta,j}, b_{\delta,j}]$, i.e.

$$C_\delta(\mu(\cdot)) = \cup_{i=1}^{k_\delta} [a_{\delta,j}, b_{\delta,j}];$$

(2) $C_1(\mu(\cdot)) \neq \phi$;

A function $\mu(\cdot)$ fulfilling conditions (1) and (2) is called *characterizing function* of the non-precise number x^*.

Remark 1. (Zimmermann, 1984). Special non-precise numbers are called *fuzzy numbers*. For them the δ-cuts are all non-empty compact intervals.

Definition 3. (Minimum Combination Rule: Zimmermann, 1984). Let $\mu_1, \mu_2, ..., \mu_n$ be fuzzy sets on $\mathbb{R}$. Then, $\zeta = \mu_1 \times \mu_2 \times ... \times \mu_n$ is a fuzzy set with:

$$\zeta(x_1, x_2, ..., x_n) = \min(\mu_1(x_1), \mu_2(x_2), ..., \mu_n(x_n)) \quad , \quad for\ all\ x_i \in \mathbb{R},\ i = 1, 2, ..., n.$$

Definition 4. (Extension Principle: Zadeh, 1965). Let $g : \Omega \longrightarrow Y$ be a function. Then g induces a function $G : F(\Omega) \longrightarrow F(Y)$, defined by

$$G(\mu)(y) = sup_{y=g(x)}\mu(x), \quad \mu \in F(\Omega),$$

where the supremum over the empty set is taken to be zero.

Now, let us consider n non-precise numbers $x_1^*, x_2^*, ..., x_n^*$. In a general setting, these n non-precise will have different characterizing functions denoted by $\mu_1(.), \mu_2(.)..., \mu_n(.)$. By the Minimum Combination Rule, it is possible to combine these n non-precise numbers into an n-dimensional fuzzy vector $\mathbf{x}^*$ with characterizing function as follows

$$\zeta(x_1, x_2, ..., x_n) = \min(\mu_1(x_1), \mu_2(x_2), ..., \mu_n(x_n)) \quad , \quad for\ all\ (x_1, x_2, ..., x_n) \in \mathbb{R}^n.$$

We note that the function $\zeta : \mathbb{R}^n \longrightarrow [0, 1]$ has the following properties:

(a) $0 \leq \zeta(x_1, x_2, ..., x_n) = \zeta(\mathbf{x}) \leq 1 \quad for\ all\ (x_1, x_2, ..., x_n) \in \mathbb{R}^n$,

(b) $\exists\ \mathbf{x}_0 \in \mathbb{R}^n$ such that $\zeta(\mathbf{x}_0) = 1$.

Let us denote the δ-cut of $\zeta(\mathbf{x}^*)$ by $C_\delta(\mathbf{x}^*)$ and define

$$C_\delta(\mathbf{x}^*) := \{x \in \mathbb{R}^n | \zeta(x) \geq \delta\},$$

then,

(c) $C_\delta(\mathbf{x}^*)$ is a closed compact and convex subset of $\mathbb{R}^n$. Moreover, for all $\delta \in (0, 1]$ the δ-cuts $C_\delta(\mathbf{x}^*)$ are Cartesian products of the δ-cuts of the n non-precise numbers $x_1^*, x_2^*, ..., x_n^*$, i.e.

$$C_\delta(\mathbf{x}^*) = C_\delta(x_1^*) \times C_\delta(x_2^*) \times ... \times C_\delta(x_n^*), \quad for\ all\ \delta(0, 1].$$

If we consider a real-valued continuous function $g(\cdot, \cdot, ..., \cdot)$ which is applied to the non-precise number $x_1^*, x_2^*, ..., x_n^*$, then the resulting value $g(x_1^*, x_2^*, ..., x_n^*)$ is again a non-precise number, denoted by y^*. Let us denote the values of the characterizing function for y^* by $\eta(\cdot)$. It follows from the extension principle that

$$\eta(y) = \begin{cases} \sup\{\zeta(\mathbf{x}) \mid g(\mathbf{x}) = y\} & if\ g^{-1}(\{y\}) \neq 0, \\ \\ 0 & if\ g^{-1}(\{y\}) = 0. \end{cases}$$

Indeed, the function $\eta(\cdot)$ of y^* is a characterizing function whose δ-cuts are given by

$$C_\delta(\mathbf{y}^*) = [\min_{\mathbf{X}\in C_\delta(\mathbf{X}^*)} g(\mathbf{x}), \max_{\mathbf{X}\in C_\delta(\mathbf{X}^*)} g(\mathbf{x})] \quad for\ all\ \delta \in (0,1].$$

3. Classical Testing of Hypotheses Problem

Let X be a random variable where it has the probability density function (p.d.f.) or the probability mass function (p.m.f.) $f_\theta(x)$ with unknown parameter $\theta \in \Theta \subset \mathbb{R}$. Usually, it will be assumed that the functional form of f_θ is known. A problem of hypothesis testing may be regarded as a decision problem where decisions have to be made about the truth of two propositions, the null hypothesis $H_0 : \theta \in \Theta_0 \subset \Theta$ and the alternative $H_1 : \theta \in \Theta_0^C = \Theta - \Theta_0$. The decision is based on an observation $\mathbf{x} = (x_1, x_2, ..., x_n)$ from a random sample $\mathbf{X} = (X_1, X_2, ..., X_n)$. In usual, such a test is dependent on values of a test statistic $T = g(\mathbf{X})$ which is evaluated for the sample, resulting in the value $t = g(\mathbf{x})$. In such problems every Borel-measurable mapping $\varphi : \mathbb{R}^n \longrightarrow [0,1]$ is known as a testing function. The power function of $\varphi(\mathbf{X})$ is defined as follows

$$\beta_\varphi(\theta) = E_\theta[\varphi(\mathbf{X})] = P_\theta\{reject\ H_0\}.$$

A test φ is said to be a test of significance level $\alpha \in [0,1]$ if $\alpha_\varphi \leq \alpha$, where

$$\alpha_\varphi = \sup_{\theta\in\Theta_0} \beta_\varphi(\theta).$$

If the test is a non-randomized test, then the space of possible values of the test statistic T is decomposed into a rejection region R and its complement R^C, the acceptance region. Depending on the hypotheses H_0 and H_1, the rejection region R takes one of the forms:

$$(a)\ T \leq t_l, \quad (b)\ T \geq t_u, \quad (c)\ T \overline{\in} (t_1, t_2),$$

where t_l, t_u or t_1 and t_2 are certain quantiles for the distribution of T such that $\alpha = \sup_{\theta\in\Theta_0} P_\theta\{reject\ H_0\}$. In case (c), we usually obtain t_1 and t_2 by the equal tails method such that $\frac{\alpha}{2} = P_\theta(T \leq t_1) = P_\theta(T \geq t_2)$. The hypothesis H_0 is rejected if the value $t = g(\mathbf{x})$ falls into the rejection region (Filzmoser and Viertl, 2004).

Definition 5. (Mood et al., 1977). Let $\{f_\theta | \theta \in \Theta\}$ be a family of p.d.f.s (p.m.f.s), $\theta \subset \mathbb{R}$. Then we say that $\{f_\theta\}$ has a MLR in the statistic $T = g(\mathbf{X})$ if for $\theta_1 < \theta_2$ (f_{θ_1} and f_{θ_2} are distinct) the ratio of $\frac{f_{\theta_2}}{f_{\theta_1}}$ is a non-decreasing function of $t = g(\mathbf{x})$ for the set of values $\mathbf{x}$ for which at least one of the f_{θ_1} and f_{θ_2} is positive (see Parchami et al., 2010).

An alternative approach for hypotheses testing which is more practical than the above method is the approach based on the p-value (Knight, 2000).

Definition 6. (Mood et al., 1977). Consider a family of test functions $\{\varphi_\alpha\}_{\alpha\in(0,1)}$ where the test function φ_α has level α. Let $\{\varphi_\alpha\}_{\alpha\in(0,1)}$ be the test functions such that $\phi_{\alpha_1}(\mathbf{x}) = 1$ implies $\varphi_{\alpha_2}(\mathbf{x}) = 1$, for any $\alpha_1 < \alpha_2$, then the p-value (or observed significance level) is defined to be $p - value = \inf\{\alpha|\varphi_\alpha(\mathbf{x}) = 1\}$.

From Definition 3, the p-value for the cases (1.a), (1.b) and (1.c) can be obtained in the following:

(a) $f(\theta_0) = P_{\theta_0}(T \leq t)$,

(b) $f(\theta_0) = P_{\theta_0}(T \geq t)$,

(c) $f(\theta_0) = 2\min[P_{\theta_0}(T \leq t), P_{\theta_0}(T \geq t)] = \begin{cases} 2P_{\theta_0}(T \geq t) & if\ t \geq m \\ 2P_{\theta_0}(T \leq t) & if\ t \leq m \end{cases}$

where θ_0 is the boundary of the null hypothesis and m is the median of the distribution T.

Lemma 1. *(Parchami et al., 2010). Let $X \sim f_\theta$, $\theta \in \Theta$, where $\{f_\theta\}$ has a MLR in $T = g(\boldsymbol{X})$. Suppose that θ_0 is the boundary of null hypothesis. Then for the tests of the forms (1.a), (1.b) and (1.c) we have the following statements:*

(1.a) the p-value is a strictly decreasing function of θ_0;

(1.b) the p-value is a strictly increasing function of θ_0;

(1.c) the p-value is a strictly increasing function of θ_0 where t is bigger than the median of the distribution of T and it is a strictly increasing function at θ_0 elsewhere.

The above Lemma shows the relationship between the p-value and boundary of the null hypothesis. For details see Parchami et al. (2010).

4. Testing Fuzzy Hypotheses with Fuzzy Data

In this Section, we define some concepts, as fuzzy sets of real numbers, for modeling the extensions of simple, one sided, two sided ordinary hypotheses to the fuzzy ones.

Definition 7. (Taheri and Behboodian, 1999). Any hypothesis of the form ($\widetilde{H} : \theta$ is H) is called a Fuzzy Hypothesis, where $H : \Theta \longrightarrow [0, 1]$ is a fuzzy subset of the parameter space Θ with membership function H.

Similar to the ordinary cases we have:

Definition 8. (Taheri and Arefi, 2009). Let θ_0 be a known real number, then:

(a) Any hypothesis of the form ($H : \theta$ is approximately θ_0) is said to be a fuzzy simple hypothesis;
(b) Any hypothesis of the form ($H : \theta$ is not approximately θ_0) is said to be a fuzzy two-sided hypothesis;
(c) Any hypothesis of the form ($H : \theta$ is essentially smaller than θ_0) is said to be a fuzzy left one-sided hypothesis;
(d) Any hypothesis of the form ($H : \theta$ is essentially bigger than θ_0) is said to be a fuzzy right one-sided hypothesis.

Note that every fuzzy hypothesis ($\widetilde{H} : \theta$ is H) such as $H \in F_S(\mathbb{R})$ or $H \in F_B(\mathbb{R})$ is an *one-sided* fuzzy hypothesis. Similarly, if $H \in F_T(\mathbb{R})$ is a *simple* fuzzy hypothesis then the hypothesis in the form $H \overline{\in} F_T(\mathbb{R})$ is a *two-sided* fuzzy hypothesis.

Definition 9. (Parchami et al., 2010). The boundary of the fuzzy hypothesis $\widetilde{H}$ is a fuzzy subset of Θ with membership function H_b. It can be one of the following forms

(a) $H_b(\theta) = \begin{cases} H(\theta) & for\ \theta \leq \theta_1 \\ 0 & for\ \theta > \theta_1 \end{cases}$

(b) $H_b(\theta) = \begin{cases} H(\theta) & for\ \theta \geq \theta_1 \\ 0 & for\ \theta < \theta_1 \end{cases}$

(c) $H_b(\theta) = H(\theta)$.

Example 1. Let θ be the parameter of a binomial distribution. Then the hypothesis $\widetilde{H} : \theta = B(0, 0.3, 0.7) \in F_B(\mathbb{R})$ which is an *one-sided* fuzzy hypothesis with increasing membership function. According to the Definition 6, $H_b = T(0.3, 0.7, 0.7) \in F_T(\mathbb{R})$ is the boundary of the fuzzy hypothesis $\widetilde{H}$.

Definition 10. (Parchami et al., 2010). Let $X \sim f_\theta,\ \theta \in \Theta$, where $\{f_\theta\}$ has a

MLR in $T(x)$. For testing a fuzzy hypotheses problem with the null fuzzy hypothesis boundary H_{0b}, by extension principle we define $\mathbf{P} \in F([0,1])$ as follows

$$\mathbf{P}(p) = G(H_{0b})(p), \quad p \in [0,1],$$

where g is the function $g : \Theta \longrightarrow [0,1]$ given as in the cases (a)-(c) in (1). The fuzzy set $\mathbf{P}$ is called the fuzzy p-valued for the related fuzzy hypotheses testing problem and is denoted by $\widetilde{p}$-value.

5. Fuzzy P-value

Let $x_1^*, x_2^*, ..., x_n^*$ be fuzzy data which are to be used for a statistical test about fuzzy hypothesis H_0. According to Sections 2 and 4, the value $t^* = g(x_1^*, x_2^*, ..., x_n^*)$ of a continuous test statistic and H_{0b} (the boundary of H_0) becomes fuzzy. We denote the characterizing function of t^* by $\eta(\cdot)$ and the characterizing function of H_{0b} by $\lambda(\cdot)$. Notice that, all of the δ-cuts of $\eta(\cdot)$ and $\lambda(\cdot)$ are closed intervals as the forms $\eta_\delta[t_1(\delta), t_2(\delta)]$ and $\lambda_\delta[\theta_1(\delta), \theta_2(\delta)]$ for all $\delta \in (0,1]$.

It should be mentioned that, in Denoeux et al. (2005) and Filzmoser and Viertl (2004), the fuzziness of the p-value is a consequence of the fuzziness of the data, and also in Parchami et al. (2010), the fuzziness of the p-value is a consequence of the fuzziness of the hypotheses. In this paper we show that the fuzziness of the p-value is a consequence of the fuzziness of both data and hypotheses. Comparing to Parchami et al. (2010), let us propose the following Theorem.

Theorem 1. *Let $X \sim f_\theta,\ \theta \in \Theta$, where $\{f_\theta\}$ has a MLR at $T(x)$. In a fuzzy hypotheses testing, for any critical region of the forms (a)-(c) indicated in (1), and $\boldsymbol{P}$ given in Definition 10, the δ-cuts $\boldsymbol{P}_\delta$ of $\boldsymbol{P}$ are as follows:*

a) $\mathbf{P}_\delta = [\mathbf{P}_{\theta_2(\delta)}(T \leq t_1(\delta)),\ \mathbf{P}_{\theta_1(\delta)}(T \leq t_2(\delta))]$,

b) $\mathbf{P}_\delta = [\mathbf{P}_{\theta_1(\delta)}(T \geq t_2(\delta)),\ \mathbf{P}_{\theta_2(\delta)}(T \geq t_1(\delta))]$,

c) $$\mathbf{P}_\delta = \begin{cases} [2\mathbf{P}_{\theta_1(\delta)}(T \geq t_2(\delta)),\ 2\mathbf{P}_{\theta_2(\delta)}(T \geq t_1(\delta))] & if\ m \geq w_r, \\ 2\mathbf{P}_{\theta_2(\delta)}(T \leq t_1(\delta)),\ 2\mathbf{P}_{\theta_1(\delta)}(T \leq t_2(\delta)) & if\ m \leq w_l, \end{cases}$$

where $w_l = \inf\{w : w \in Supp(\boldsymbol{w})\}$ and $w_r = \sup\{w : w \in Supp(\boldsymbol{w})\}$ in which the fuzzy set $\boldsymbol{w}$ (with characterizing function $\boldsymbol{w}(w) = H_{0b}(\theta)$ where w is the median of the distribution $T(x)$ under θ) is called the median of the distribution for the test statistic under the fuzzy null boundary $H_{0b}(\theta)$.

Proof. Let M be boundary membership function obtained from combining $\eta(\cdot)$ and $\lambda(\cdot)$. From the extension principle, in the cases (1.a), (1.b) and also (1.c) where $m > w_r$ or $m < w_l$, we have

$$\mathbf{P}(p) = \sup_{p=gof(\theta,t)} M_{0,b}(\theta, t).$$

In case (1.a), if we define

$$g(x) = x \ \ and \ \ f(t_i, \theta_j) = P(T \leq t_i|\theta_j)|, \quad i \neq j, \quad i = 1, 2, \quad j = 1, 2,$$

then due to Lemma 1, the functions f and g are one to one functions. Therefore,

$$\mathbf{P}(p) = M_{0,b}(f^{-1}og^{-1}(p)), \quad p \in [0, 1].$$

One can obtain the δ-cut of $\mathbf{P}$ in the following

$$\mathbf{P}_\delta = \{p \in [0, 1] : \mathbf{P}(p) \geq \delta\}$$

$$= \{p \in [0, 1] : f^{-1}og^{-1}(p) \in [(t_1(\delta), \theta_2(\delta)), (t_2(\delta), \theta_1(\delta))]\}.$$

So, for all $\delta \in [0, 1]$, we have

$$\mathbf{P}_\delta = \{p \in [0, 1] : p \in [gof(t_1(\delta), \theta_2(\delta)), gof(t_2(\delta), \theta_1(\delta))]\},$$

$$\mathbf{P}_\delta = [P(T \leq t_1(\delta)|\theta_2(\delta)), P(T \leq t_2(\delta)|\theta_1(\delta))]$$

Also, in case (1.b) if we define

$$f(t_i, \theta_j) = P(T \geq t_i|\theta_j), \quad i \neq j, \quad i = 1, 2, \quad j = 1, 2,$$

and in case (1.c) we define

$$f(t_i, \theta_j) = \begin{cases} 2P(T > t_i \mid \theta_j) & i \neq j, \ \ i = 1, 2, \ \ j = 1, 2 \ \ if \ \ m \geq w_r \\ \\ 2P(T < t_i \mid \theta_j) & i \neq j, \ \ i = 1, 2, \ \ j = 1, 2 \ \ if \ \ m \leq w_l \end{cases}$$

the proof is complete in similar way.

The Theorem 1 is proved.

Example 2. A petroleum company believes that produce petrol with octane about 87 and variance $\delta = 4$. To examine this claim, a random sample of 25 from this company is selected. It is observed that octane is approximately 90. Whether the claim can be accepted in the level 0.95 confidence?

Solution. Let us consider hypotheses as follows:

$$\widetilde{H}_0 : \mu \ is \ approximately \ 87$$

$$\widetilde{H}_1 : \mu \ is \ approximately \ bigger \ than \ 87$$

which have membership function $\widetilde{H}_0$ and $\widetilde{H}_1$. Assuming

$$B(86, 88) \in F_B(R), \quad T(85, 87, 89) \in F_T(R),$$

and also membership function "approximately 90" $T(88, 90, 92) \in F_T(R)$.

According to the case (b) from Theorem 1 we have the following:

$$\widetilde{P}_\delta = [\int_{\frac{t_2(\delta)-\theta_1(\delta)}{\frac{2}{\sqrt{25}}}}^{\infty} (2\pi)^{-\frac{1}{2}} \exp(-\frac{z^2}{2}) dz, \int_{\frac{t_1(\delta)-\theta_2(\delta)}{\frac{2}{\sqrt{25}}}}^{\infty} (2\pi)^{-\frac{1}{2}} \exp(-\frac{z^2}{2}) dz].$$

Let $H_{0b} = T(85, 87, 89)$ be boundary point. Therefore,

$$\theta_1(\delta) = 85 + 2\delta, \quad \theta_2(\delta) = 89 - 2\delta, \quad for \ all \ \delta \in (0, 1],$$

$$t_1(\delta) = 88 + 2\delta, \quad t_2(\delta) = 92 - 2\delta, \quad for \ all \ \delta \in (0, 1],$$

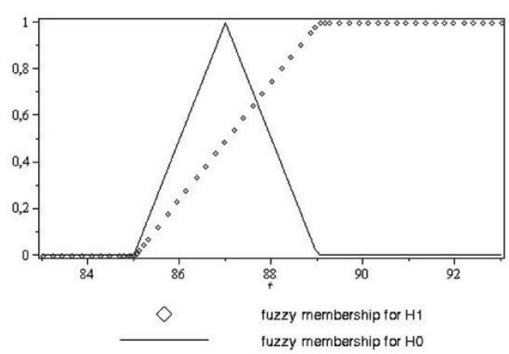

Figure 1: Membership functions H_0 and H_1

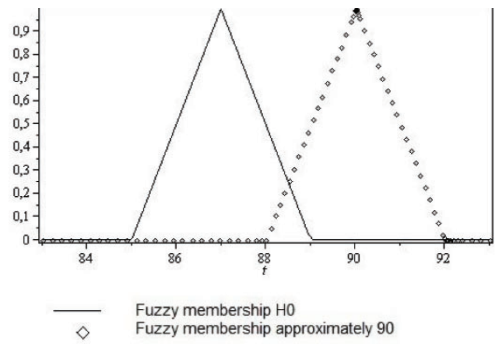

Figure 2: Membership functions of H_0 and approximately 90

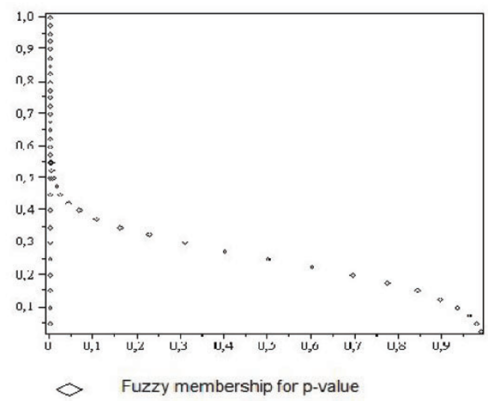

Figure 3: Membership functions fuzzy p-value

As we see from Figure 3, $0 < p - value < 0.98$. So, according to the fact that significance level (α) with respect to *p-value* in what situation is, $\widetilde{H}_0$ is rejected or accepted.

Example 3. A manufacturer of prefabricated parts claims that products resistance is normal for which standard deviation of products resistance is 20 kg/cm^2. A random sample of 25 from these products selected and the average results were approximately 330. We are going to test these hypotheses about this company:

$$\widetilde{H}_0 : \mu \ nearly \ 350,$$

$$\widetilde{H}_1 : \mu \ beyond \ 350.$$

Solution. Let $T(345, 350, 355) \in F_T(R)$ and $\widetilde{H}_1 = 1 - \widetilde{H}_0$ be membership functions for $\widetilde{H}_0$ and $\widetilde{H}_1$, respectively. Let also $T(320, 330, 340) \in F_T(R)$ be "approximately 330" membership function (Figure 5). So, due to the case (c) from Theorem 1, since $m \geq w_r$, $(m = 350,\ w_r = 340)$, hence δ-cuts of the Fuzzy p-value can be defined in the following:

$$\widetilde{P}_\delta = [2\int_{\frac{t_2(\delta)-\theta_1(\delta)}{\frac{2}{\sqrt{25}}}}^{\infty}(2\pi)^{-\frac{1}{2}}\exp(-\frac{z^2}{2})dz, 2\int_{\frac{t_1(\delta)-\theta_2(\delta)}{\frac{2}{\sqrt{25}}}}^{\infty}(2\pi)^{-\frac{1}{2}}\exp(-\frac{z^2}{2})dz].$$

In addition, supposing $H_{0b} = T(345, 350, 355)$ is boundary point for $\widetilde{H}_0$. Therefore,

$$\theta_1(\delta) = 345 + 5\delta, \quad \theta_2(\delta) = 355 - 5\delta, \quad for\ all\ \delta \in (0, 1],$$

$$t_1(\delta) = (10)\delta + 320, \quad t_2(\delta) = (-10)\delta + 340, \quad for\ all\ \delta \in (0, 1],$$

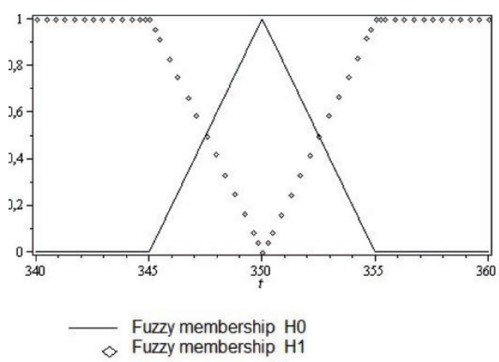

Figure 4: Membership functions H_0 and H_1

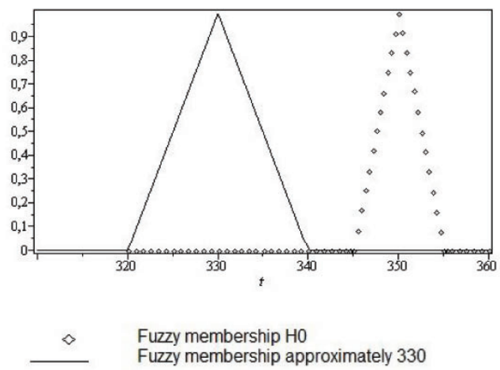

Figure 5: Membership functions H_0 and approximately 330

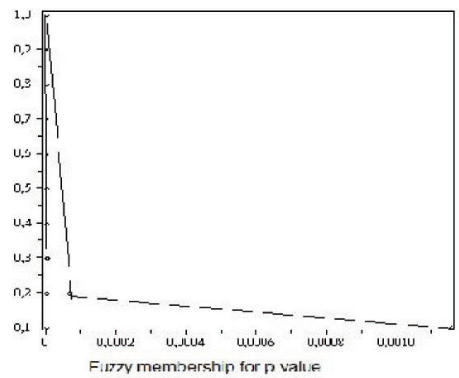

Figure 6: Membership function fuzzy p-value

As we see from Figure 6, $p - value < 0.5$. So, the hypothesis $\widetilde{H}_0$ is rejected.

6. Conclusion

The proposed method in this paper includes all cases, for instance the classical case. In this paper, if the data or hypotheses or both are simple, then it is enough to be placed just numbers in which the previous contents will be performed. Also, we have given numerical examples to show the performance of the proposed method.

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