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EXISTENCE OF NONOSCILLATORY BOUNDED SOLUTIONS FOR A SYSTEM OF SECOND-ORDER NONLINEAR NEUTRAL DELAY DIFFERENTIAL EQUATIONS

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Abstract. A system of second-order nonlinear neutral delay differential equations

$$\Big(r_1(t)\big(x_1(t)+P_1(t)x_1(t-\tau_1)\big)'\Big)'=F_1\big(t,x_2(t-\sigma_1),x_2(t-\sigma_2)\big),$$
$$\Big(r_2(t)\big(x_2(t)+P_2(t)x_2(t-\tau_2)\big)'\Big)'=F_2\big(t,x_1(t-\sigma_1),x_1(t-\sigma_2)\big),$$

where $\tau_i>0,\sigma_1,\sigma_2\geq 0, r_i\in C([t_0,+\infty),\mathbb{R}^+), P_i(t)\in C([t_0,+\infty),\mathbb{R}), F_i\in C([t_0,+\infty)\times\mathbb{R}^2,\mathbb{R}), i=1,2$ is studied in this paper, and some sufficient conditions for existence of nonoscillatory bounded solutions for this system are established by Krasnoselkii and Schauder fixed point theorems, and expressed through several theorems according to the range of the value of the functions $P_1(t),P_2(t)$ and their combination.

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1. Introduction and preliminaries

We investigate the following nonlinear differential system

$$\Big(r_1(t)\big(x_1(t)+P_1(t)x_1(t-\tau_1)\big)'\Big)'=F_1\big(t,x_2(t-\sigma_1),x_2(t-\sigma_2)\big),$$
$$\Big(r_2(t)\big(x_2(t)+P_2(t)x_2(t-\tau_2)\big)'\Big)'=F_2\big(t,x_1(t-\sigma_1),x_1(t-\sigma_2)\big),$$

which may be rewritten as

$$\Big(r_i(t)\big(x_i(t)+P_i(t)x_i(t-\tau_i)\big)'\Big)'=F_i\big(t,x_{3-i}(t-\sigma_1),x_{3-i}(t-\sigma_2)\big),\quad t\geq t_0, \tag{1.1}$$

where $\tau_i>0,\sigma_1,\sigma_2\geq 0, r_i\in C([t_0,+\infty),\mathbb{R}^+), P_i(t)\in C([t_0,+\infty),\mathbb{R}), F_i\in C([t_0,+\infty)\times\mathbb{R}^2,\mathbb{R})$ and $i=1,2$.

By applying Krasnoselkii and Schauder fixed point theorems and some new techniques, we obtained a few sufficient conditions for the existence of a nonoscillatory bounded solution of the system (1.1).

Lemma 1.1(Krasnoselskii Fixed Point Theorem)[4]*Let Ω be a bounded closed convex subset of a Banach space X and $Q, S : \Omega \to X$ satisfy $Qx + Sy \in \Omega$ for each $x, y \in \Omega$. If Q is a contraction mapping and S is a completely continuous mapping, then the equation $Qx + Sx = x$ has at least one solution in Ω.*

Lemma 1.2(Schauder Fixed Point Theorem)[4]*Let Ω be a closed, convex and nonempty subset of a Banach space X and $S : \Omega \to \Omega$ be a continuous mapping such that $S\Omega$ is a relatively compact subset of X. Then S has at least one fixed point in Ω. That is there exists an $x \in \Omega$ such that $Sx = x$.*

2. Existence of nonoscillatory bounded solutions

In this section, a few sufficient conditions of the existence of nonoscillatory bounded solutions for system (1.1) will be given.

Theorem 2.1 *Let functions $h_i, q_i, r_i \in C([t_0, +\infty), \mathbb{R}^+)$ and $P_i(t) \in C([t_0, +\infty), \mathbb{R})$ satisfy that*

$$0 < P_i(t) \le P_i < 1, \tag{2.1}$$

$$\big|F_i(t, u_1, u_2) - F_i(t, v_1, v_2)\big| \le h_i(t) \max\big\{|u_i - v_i| : \ i = 1, 2\big\}, \tag{2.2}$$

$$\big|F_i(t, u_1, u_2)\big| \le q_i(t), \tag{2.3}$$

$$\int_{t_0}^{+\infty} R_i(t) \max\big\{h_i(t), q_i(t)\big\} dt < +\infty, \tag{2.4}$$

where $R_i(t) = \int_{t_0}^{t} \frac{1}{r_i(s)} ds$ and $i = 1, 2$. Then the system (1.1) has a nonoscillatory bounded solution.

Proof. In virtue of (2.4), a sufficiently large $T > t_0$ can be chosen such that

$$\int_{T}^{+\infty} R_i(t) \max\big\{h_i(t), q_i(t)\big\} dt < \frac{1 - P_i}{4}, \tag{2.5}$$

where $i = 1, 2$.

Let $C([t_0, +\infty), \mathbb{R}^2)$ be the set of all continuous vector functions $x(t) = (x_1(t), x_2(t))$ with the norm $||x|| = \sup_{t \ge t_0}\{|x_1(t)|, |x_2(t)|\} < +\infty$. Obviously, $C([t_0, +\infty), \mathbb{R}^2)$ is a Banach space. Now, define a bounded, closed and convex subset Ω of $C([t_0, +\infty), \mathbb{R}^2)$ as following:

$$\Omega = \Big\{x = (x_1, x_2) \in C([t_0, +\infty), \mathbb{R}^2) : \frac{1 - P_i}{2} \le x_i(t) \le 1, i = 1, 2, t \ge t_0\Big\}.$$

Let mappings $Q = (Q_1, Q_2)$ and $S = (S_1, S_2) : \Omega \to C([t_0, +\infty), \mathbb{R}^2)$ be defined by

$$(Q_i x)(t) = \begin{cases} \frac{3-P_i}{4} - P_i(t)x_i(t-\tau_i) \\ \qquad - \int_T^t R_i(s)F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2))ds, & t \geq T \\ (Q_i x)(T), & t_0 \leq t < T \end{cases} \tag{2.6}$$

$$(S_i x)(t) = \begin{cases} -R_i(t)\int_t^{+\infty} F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2))ds, & t \geq T \\ (S_i x)(T), & t_0 \leq t < T \end{cases} \tag{2.7}$$

for all $x \in \Omega$, where $i = 1, 2$.

(i) It is claimed that $Qx + Sy \in \Omega$ for all $x, y \in \Omega$, i.e. $Q\Omega \cup S\Omega \subset \Omega$.

In fact, for each $x, y \in \Omega$ and $t \geq T$, it follows from (2.3) and (2.5) that

$$\begin{aligned}(Q_i x)(t) + (S_i y)(t) \geq &\frac{3+P_i}{4} - P_i x_i(t-\tau_i) \\ &- \int_T^t R_i(s)|F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2))|ds \\ &- R_i(t)\int_t^{+\infty} |F_i(s, y_{3-i}(s-\sigma_1), y_{3-i}(s-\sigma_2))|ds \\ \geq &\frac{3+P_i}{4} - P_i - \int_T^t R_i(s)q_i(s)ds - \int_t^{+\infty} R_i(s)q_i(s)ds \\ \geq &\frac{3+P_i}{4} - P_i - \int_T^{+\infty} R_i(s)q_i(s)ds \\ \geq &\frac{1-P_i}{2},\end{aligned}$$

and

$$(Q_i x)(t) + (S_i y)(t) \leq \frac{3+P_i}{4} + 0 + \int_T^{+\infty} R_i(s)q_i(s)ds \leq 1.$$

Thus, $\frac{1-P_i}{2} \leq (Q_i x)(t) + (S_i y)(t) \leq 1, i = 1, 2$ for $t \geq t_0$.

(ii) It is declared that Q is a contraction mapping on Ω.

In reality, for any $x, y \in \Omega$ and $t \geq T$, it is easy to derive that

$$\begin{aligned}
&\big|(Q_ix)(t) - (Q_iy)(t)\big| \\
&\quad \leq P_i(t)\big|x_i(t-\tau_i) - y_i(t-\tau_i)\big| + \int_T^t R_i(s) \\
&\qquad \big|F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2)) - F_i(s, y_{3-i}(s-\sigma_1), y_{3-i}(s-\sigma_2))\big| ds \\
&\quad \leq P_i\big|x_i(t-\tau_i) - y_i(t-\tau_i)\big| \\
&\qquad + \int_T^t R_i(s)h_i(s)\max\big\{|x_{3-i}(s-\sigma_j) - y_{3-i}(s-\sigma_j)| : j = 1, 2\big\} ds \\
&\quad \leq P_i||x-y|| + \int_T^{+\infty} R_i(s)h_i(s)ds||x-y|| \\
&\quad \leq k_i||x-y||,
\end{aligned}$$

which implies that

$$||Q_ix - Q_iy|| \leq k_i||x-y||.$$

It follows from $k_i = P_i + \int_T^{+\infty} R_i(s)h_i(s)ds \leq \frac{1+3P_i}{4} < 1$ that Q is a contraction mapping on Ω.

(iii) It can be asserted that S is completely continuous.

Firstly, we show S is continuous. Let $x_k = (x_{1k}(t), x_{2k}(t)) \in \Omega$ and $x_{ik}(t) \to x_i(t)$ as $k \to +\infty$. Since Ω is closed, $x = (x_1(t), x_2(t)) \in \Omega$. For $t \geq T$, (2.2) guarantees that

$$\begin{aligned}
&\big|(S_ix_k)(t) - (S_ix)(t)\big| \\
&\leq R_i(t)\int_t^{+\infty} \big|F_i(s, x_{3-i\ k}(s-\sigma_1), x_{3-i\ k}(s-\sigma_2)) - F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2))\big| ds \\
&\leq \int_t^{+\infty} R_i(s)h_i(s)\max\big\{|x_{3-i\ k}(s-\sigma_j) - x_{3-i}(s-\sigma_j)| : j = 1, 2\big\} ds \\
&\leq \|x_k - x\| \int_T^{+\infty} R_i(s)h_i(s)ds.
\end{aligned}$$

This above inequality together with (2.4) implies that S is continuous.

Next, we prove $S\Omega$ is relatively compact. It is sufficient to show that the family of functions $\{Sx : x \in \Omega\}$ is uniformly bounded and equicontinuous on $[t_0, +\infty)$. The uniform boundedness is obvious. For the equicontinuity, according to Levitan's result [10], it is only need to prove that, for any given $\varepsilon > 0$, $[t_0, +\infty)$ can be decomposed into finite subintervals in such a way that on each subinterval all functions of the family have change of amplitude less than ε. By (2.4), for any $\varepsilon > 0$, take $T' \geq T$ large enough so that

$$\int_{T'}^{+\infty} R_i(s)q_i(s)ds < \frac{\varepsilon}{2}. \tag{2.8}$$

Then, for any $x \in \Omega$ and $t_2 > t_1 \geq T'$, (2.8) ensures that

$$\begin{aligned}\big|(S_ix)(t_2) - (S_ix)(t_1)\big| \leq & R_i(t)\int_{t_2}^{+\infty}\big|F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2))\big|ds \\ & + R_i(t)\int_{t_1}^{+\infty}\big|F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2))\big|ds \\ \leq & \int_{T'}^{+\infty} R_i(s)q_i(s)ds + \int_{T'}^{+\infty} R_i(s)q_i(s)ds \\ < & \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.\end{aligned}$$

For any $x \in \Omega$ and $T \leq t_1 < t_2 \leq T'$, there exists $\delta > 0$ such that if $0 < t_2 - t_1 < \delta$, then

$$\begin{aligned}\big|(S_ix)(t_2) - (S_ix)(t_1)\big| \leq & R_i(t)\int_{t_1}^{t_2}\big|F_i(s, x_{3-i}(s-\sigma_1), x_{3-i}(s-\sigma_2))\big|ds \\ \leq & \int_{t_1}^{t_2} R_i(s)q_i(s)ds < \varepsilon.\end{aligned}$$

For any $x \in S$ and $t_0 \leq t_1 < t_2 \leq T$, it is easy to get that

$$\big|(S_ix)(t_2) - (S_ix)(t_1)\big| = 0 < \varepsilon.$$

Consequently, $\{S_ix : x \in \Omega\}$ is uniformly bounded and equicontinuous on $[t_0, +\infty)$. Therefore $S\Omega$ is relatively compact. It follows from Lemma 1.1 that there is $x_0 \in \Omega$ such that $Qx_0 + Sx_0 = x_0$. Obviously, $x_0(t)$ is a nonoscillatory bounded solution of the system (1.1). This completes the proof.

Theorem 2.2 *Let functions $h_i, q_i, r_i \in C([t_0, +\infty), \mathbb{R}^+)$ and $P_i(t) \in C([t_0, +\infty), \mathbb{R})$ satisfy (2.2)∼(2.4) and*

$$1 < a_i \leq P_i(t) \leq b_i < +\infty, \tag{2.9}$$

where $i = 1, 2$. Then the system (1.1) has a nonoscillatory bounded solution.

Proof. By (2.4), a sufficiently large $T > t_0$ can be chosen such that

$$\int_T^{+\infty} R_i(t)h_i(t)dt < a_i - 1, \tag{2.10}$$

$$\int_T^{+\infty} R_i(t)q_i(t)dt < \frac{a_ib_i - a_i^2 - b_i}{2}, \tag{2.11}$$

where $i = 1, 2$.

Let $C([t_0,+\infty),\mathbb{R}^2)$ be the set as in the proof of Theorem 2.1 and define a bounded, closed and convex subset Ω of $C([t_0,+\infty),\mathbb{R}^2)$ as following:

$$\Omega=\Big\{x=(x_1,x_2)\in C([t_0,+\infty),\mathbb{R}^2): a_i\le x_i(t)\le b_i, i=1,2, t\ge t_0\Big\}.$$

Let mappings $Q=(Q_1,Q_2)$ and $S=(S_1,S_2):\Omega\to C([t_0,+\infty),\mathbb{R}^2)$ be defined as

$$(Q_ix)(t)=\begin{cases}\frac{a_i^2+a_ib_i+b_i}{2a_i}-\frac{x_i(t+\tau_i)}{P_i(t+\tau_i)}-\frac{1}{P_i(t+\tau_i)}\\ \qquad\int_T^{t+\tau_i}R_i(s)F_i(s,x_{3-i}(s-\sigma_1),x_{3-i}(s-\sigma_2))ds, & t\ge T\\ (Q_ix)(T), & t_0\le t<T\end{cases}\tag{2.12}$$

$$(S_ix)(t)=\begin{cases}-\frac{R_i(t+\tau_i)}{P_i(t+\tau_i)}\int_{t+\tau_i}^{+\infty}F_i(s,x_{3-i}(s-\sigma_1),x_{3-i}(s-\sigma_2))ds, & t\ge T\\ (S_ix)(T), & t_0\le t<T\end{cases}\tag{2.13}$$

for all $x\in\Omega$, where $i=1,2$.

Proceeding similarly as what we did in Theorem 2.1, we prove that the system (1.1) has a nonoscillatory bounded solution. The proof is completed.

Theorem 2.3 *Let functions $h_i,q_i,r_i\in C([t_0,+\infty),\mathbb{R}^+)$ and $P_i(t)\in C([t_0,+\infty),\mathbb{R})$ satisfy (2.2)∼(2.4),*

$$0<P_1(t)\le P_1<1,\tag{2.14}$$

and

$$1<a_2\le P_2(t)\le b_2<+\infty.\tag{2.15}$$

Then the system (1.1) has a nonoscillatory bounded solution.

Proof. By (2.4), a sufficiently large $T>t_0$ can be chosen such that

$$\int_T^{+\infty}R_1(t)\max\big\{h_1(t),q_1(t)\big\}dt<\frac{1-P_1}{4},\tag{2.16}$$

$$\int_T^{+\infty}R_2(t)h_2(t)dt<a_2-1,\tag{2.17}$$

$$\int_T^{+\infty}R_2(t)q_2(t)dt<\frac{a_2b_2-a_2^2-b_2}{2}.\tag{2.18}$$

Let $C([t_0,+\infty),\mathbb{R}^2)$ be the set as in the proof of Theorem 2.1 and define a bounded, closed and convex subset Ω of $C([t_0,+\infty),\mathbb{R}^2)$ as following:

$$\Omega=\Big\{x=(x_1,x_2)\in C([t_0,+\infty),\mathbb{R}^2):\frac{1-P_1}{2}\le x_1(t)\le 1, a_2\le x_2(t)\le b_2, t\ge t_0\Big\}.$$

Let mappings $Q = (Q_1, Q_2)$ and $S = (S_1, S_2) : \Omega \to C([t_0, +\infty), \mathbb{R}^2)$ be defined as

$$(Q_1x)(t) = \begin{cases} \frac{3-P_1}{4} - P_1(t)x_1(t-\tau_1) \\ \qquad - \int_T^t R_1(s)F_1(s, x_2(s-\sigma_1), x_2(s-\sigma_2))ds, & t \geq T \\ (Q_1x)(T), & t_0 \leq t < T \end{cases} \tag{2.19}$$

$$(S_1x)(t) = \begin{cases} -R_1(t)\int_t^{+\infty} F_1(s, x_2(s-\sigma_1), x_2(s-\sigma_2))ds, & t \geq T \\ (S_1x)(T), & t_0 \leq t < T \end{cases} \tag{2.20}$$

$$(Q_2x)(t) = \begin{cases} \frac{a_2^2+a_2b_2+b_2}{2a_2} - \frac{x_2(t+\tau_2)}{P_2(t+\tau_2)} - \frac{1}{P_2(t+\tau_2)} \\ \qquad \int_T^{t+\tau_2} R_2(s)F_2(s, x_1(s-\sigma_1), x_1(s-\sigma_2))ds, & t \geq T \\ (Q_2x)(T), & t_0 \leq t < T \end{cases} \tag{2.21}$$

$$(S_2x)(t) = \begin{cases} -\frac{R_2(t+\tau_2)}{P_2(t+\tau_2)}\int_{t+\tau_2}^{+\infty} F_2(s, x_1(s-\sigma_1), x_1(s-\sigma_2))ds, & t \geq T \\ (S_2x)(T), & t_0 \leq t < T \end{cases} \tag{2.22}$$

for all $x \in \Omega$.

Proceeding similarly as in the proof of Theorem 2.1 and 2.2, we obtain that the system (1.1) has a nonoscillatory bounded solution. This completes the proof.

Theorem 2.4 *Let functions $h_i, q_i, r_i \in C([t_0, +\infty), \mathbb{R}^+)$ and $P_i(t) \in C([t_0, +\infty), \mathbb{R})$ satisfy (2.2), (2.3),*

$$P_i(t) \equiv -1, \tag{2.23}$$

and

$$\int_{t_0}^{+\infty} t|R_i'(t)| \int_t^{+\infty} \max\{q_i(s), h_i(s)\}dsdt < +\infty, \tag{2.24}$$

where $i = 1, 2$. Then the system (1.1) has a nonoscillatory bounded solution.

Proof. According to a known result (Theorem 3.2.6 in [4]), (2.24) is equivalent to the condition

$$\sum_{j=0}^{\infty} \int_{t_0+j\tau_i}^{+\infty} |R_i'(t)| \int_t^{+\infty} \max\{q_i(s), h_i(s)\}dsdt < +\infty, \quad i = 1, 2. \tag{2.25}$$

By (2.25), a sufficiently large $T > t_0$ can be chosen such that

$$\sum_{j=1}^{\infty} \int_{T+j\tau_i}^{+\infty} |R_i'(t)| \int_t^{+\infty} \max\{q_i(s), h_i(s)\}dsdt < 1, \quad i = 1, 2. \tag{2.26}$$

Let $C([t_0, +\infty), \mathbb{R}^2)$ be the set as in the proof of Theorem 2.1 and define a bounded, closed and convex subset Ω of $C([t_0, +\infty), \mathbb{R}^2)$ as following:

$$\Omega = \Big\{x = (x_1, x_2) \in C([t_0, +\infty), \mathbb{R}^2) : 1 \leq x_i(t) \leq 3, i = 1, 2, t \geq t_0\Big\}.$$

Let mapping $S = (S_1, S_2) : \Omega \to C([t_0, +\infty), \mathbb{R}^2)$ be defined as

$$(S_i x)(t) = \begin{cases} 2 - \sum_{j=1}^{\infty} \int_{t+j\tau_i}^{+\infty} R_i'(s) \\ \qquad \int_s^{+\infty} F_i(u, x_{3-i}(u-\sigma_1), x_{3-i}(u-\sigma_2)) duds, & t \geq T \\ (S_i x)(T), & t_0 \leq t < T \end{cases} \tag{2.27}$$

for all $x \in \Omega$, where $i = 1, 2$.

Similarly to the proof in Theorem 2.1, we get that $S\Omega \subset \Omega$, S is a continuous mapping on Ω, and $S\Omega$ is a relatively compact subset. Applying Lemma 1.2, we could find a $x_0 = (x_{01}, x_{02}) \in \Omega$ such that $Sx_0 = x_0$. That is

$$x_{0i}(t) = \begin{cases} 2 - \sum_{j=1}^{\infty} \int_{t+j\tau_i}^{+\infty} R_i'(s) \\ \qquad \int_s^{+\infty} F_i(u, x_{0\ 3-i}(u-\sigma_1), x_{0\ 3-i}(u-\sigma_2)) duds, & t \geq T \\ x_{0i}(T), & t_0 \leq t < T \end{cases} \tag{2.28}$$

where $i = 1, 2$. For $t \geq T$,

$$x_{0i}(t) - x_{0i}(t-\tau_i) = \int_t^{+\infty} R_i'(s) \int_s^{+\infty} F_i(u, x_{0\ 3-i}(u-\sigma_1), x_{0\ 3-i}(u-\sigma_2)) duds.$$

Then,

$$\big(x_{0i}(t) - x_{0i}(t-\tau_i)\big)' = -R_i'(t) \int_t^{+\infty} F_i(u, x_{0\ 3-i}(u-\sigma_1), x_{0\ 3-i}(u-\sigma_2)) du,$$

which we can rewrite it as

$$r_i(t)\big(x_{0i}(t) + P_i(t)x_{0i}(t-\tau_i)\big)' = -\int_t^{+\infty} F_i(u, x_{0\ 3-i}(u-\sigma_1), x_{0\ 3-i}(u-\sigma_2)) du.$$

Finding the derivative,

$$\Big(r_i(t)\big(x_{0i}(t) + P_i(t)x_{0i}(t-\tau_i)\big)'\Big)' = F_i\big(t, x_{0\ 3-i}(t-\sigma_1), x_{0\ 3-i}(t-\sigma_2)\big).$$

Therefore, $x_0(t)$ is a bounded nonoscillatory solution of the system (1.1). This completes the proof.

Remark 2.5 Proceeding as before, we can prove that no matter $P_i(t)$ belongs to which cases:
(1) $0 < P_i(t) \leq P_i < 1$,
(2) $1 < a_i \leq P_i(t) \leq b_i < +\infty$,
(3) $P_i(t) \equiv -1$,
(4) $-1 < P_i \leq P_i(t) < 0$,

(5) $-\infty < a_i \leq P_i(t) \leq b_i < -1$,
(6) any combination of the above,
the system (1.1) has a bounded nonoscillatory solution.

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