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WEAKLY PAIRWISE B-IRRESOLUTE FUNCTIONS

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Abstract. As a generalization of b-irresolute functions, we introduce the notion of weakly b-irresolute functions in bitopological spaces and obtain several characterizations and some properties of weakly b-irresolute functions.

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1. Introduction

The concept of bitopological spaces was first introduced by Kelly [0]. After the introduction of the definition of a bitopological space by Kelly, a large number of topologists have turned their attention to the generalization of different concepts of a single topological space in this space. In the present paper, we introduce the notion of weakly b-irresolute functions in bitopological spaces and obtain several characterizations and some properties of weakly b-irresolute functions. Throughout this paper, the triple (X, τ_1, τ_2) where X is a set and τ_1 and τ_2 are topologies on X, will always denote a bitopological space. Let (X, τ_1, τ_2) be a bitopological space and A a subset of X. The closure of A and the interior of A with respect to τ_i are denoted by $i(A)$ and $i(A)$, respectively, for $i = 1, 2$.

2.Preliminaries

Definition 1. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j)-b-open [0] if $A \subset j(i(A)) \cup i(j(A))$, where $i \neq j$, $i, j = 1, 2$. We have set (i, j)-$BO(X, x)$ = $\{V \in (i, j)\text{-}BO(X) : x \in V\}$ for $x \in X$. The complement of an (i, j)-b-open set is called an (i, j)-b-closed set.*

Definition 2.[0] *The intersection (resp. union) of all (i, j)-b-closed (resp. (i, j)-b-open) sets of X containing (resp. contained in) $A \subset X$ is called the (i, j)-b-closure (resp. (i, j)-b-interior) of A and is denoted by (i, j)-$b(A)$ (resp. (i, j)-$b(A)$).*

Lemma 1.[0] *Let (X, τ_1, τ_2) be a bitopological space and A a subset of X. Then*

1. *(i, j)-$b(A)$ is (i, j)-b-open;*
2. *(i, j)-$b(A)$ is (i, j)-b-closed;*
3. *A is (i, j)-b-open if and only if $A = (i, j)$-$b(A)$;*
4. *A is (i, j)-b-closed if and only if $A = (i, j)$-$b(A)$;*
5. *(i, j)-$b(X\backslash A) = X\backslash(i, j)$-$b(A)$;*
6. *(i, j)-$b(X\backslash A) = X\backslash(i, j)$-$b(A)$.*

Lemma 2.[0] *Let (X, τ_1, τ_2) be a bitopological space and $A \subset X$. A point $x \in (i, j)$-$b(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in (i, j)$-$BO(X, x)$.*

Definition 3.[0] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-b-continuous if for each $x \in X$ and each σ_i-open set V of Y containing $f(x)$, there exists an (i, j)-b-open set U containing x such that $f(U) \subset V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be pairwise b-continuous if f is $(1, 2)$-b-continuous and $(2, 1)$-b-continuous.*

Definition 4.[0] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-b-irresolute if for each $x \in X$ and each (i, j)-b-open set V of Y containing $f(x)$, there exists an (i, j)-b-open set U containing x such that $f(U) \subset V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be pairwise b-irresolute if f is $(1, 2)$-b-irresolute and $(2, 1)$-b-irresolute.*

3. Weakly pairwise b-irresolute functions

Definition 5. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly pairwise b-irresolute if for each $x \in X$ and each (i, j)-b-open set V containig $f(x)$, there is an (i, j)-b-open set U in X such that $x \in U$ and $f(U) \subset (i, j)$-$b(V)$, $i \neq j$, i,j=1,2.*

Definition 6. *A bitopological space (X, τ_1, τ_2) is strongly s-pairwise regular, if and only if for each point $x \in X$ and each (i, j)-b-open set U such that $x \in U$ there exists an (i, j)-b-open set W such that $x \in W \subset (i, j)$-$b(W) \subset U$, $i \neq j$, i,j=1,2.*

Theorem 1. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces, and let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and the space Y is strongly s-pairwise regular, then f is weakly pairwise b-irresolute if and only if f is pairwise b-irresolute, for $i \neq j$, i,j=1,2.*

Proof. Sufficiency. Each pairwise b-irresolute function is weakly pairwise irresolute. Necessity. Let $x \in X$ and V be any (i, j)-b-open set of Y containing $f(x)$. In the strongly s-pairwise regular space Y, there exists an (i, j)-b-open set U such that $f(x) \in U \subset (i,j)$-$b(U) \subset V$. Since f is weakly pairwis b-irresolute, so there exists an (i,j)-b-open set W such that $x \in W$ and $f(W) \subset (i,j)$-$b(U) \subset V$. Hence $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise b-irresolute.

Lemma 4. *For any function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following conditions are equivalent:*

1. *For any subset A of Y, (i,j)-$b(f^{-1}[(i,j)$-$b((i,j)$-$b(A)])) \subset f^{-1}[(i,j)$-$b(A)]$.*
2. *For any (i,j)-b-open set G in Y, (i,j)-$b(f^{-1}(G)) \subset f^{-1}[(i,j)$-$b(G)]$.*
3. *For any (i,j)-b-closed set H in Y, (i,j)-$b(f^{-1}[(i,j)$-$b(H)]) \subset f^{-1}[H]$. Here $i \neq j$, i,j=1,2.*

Proof. (1)⇒(2): Suppose that G is any (i,j)-b-open set in Y. Then, by (1), (i,j)-$b(f^{-1}[(i,j)$-$b((i,j)$-$b(G)])) \subset f^{-1}[(i,j)$-$b(G)]$. Since G is (i,j)-b-open, $G \subset [(i,j)$-$b((i,j)$-$b(G)])$. Consequently, (i,j)-$b(f^{-1}(G)) \subset f^{-1}[(i,j)$-$b(G)]$.
(2)⇒(3): For any (i,j)-b-closed set H, (i,j)-$b(H)$ is (i,j)-b-open in Y. Therefore, by (2), (i,j)-$b(f^{-1}[(i,j)$-$b(H)]) \subset f^{-1}[(i,j)$-$b((i,j)$-$b(H)])$. Recall that, for the (i,j)-b-closed set H, (i,j)-$b((i,j)$-$b(H)) \subset H$. Therefore, (i,j)-$b(f^{-1}[(i,j)$-$b(H)]) \subset f^{-1}(H)$.
(3)⇒(1): Let A be any subset of Y. Let $H = (i,j)$-$b(A)$. Then for the (i,j)-b-closed set H, by (3), (i,j)-$b(f^{-1}[(i,j)$-$b((i,j)$-$b(A)]))) \subset f^{-1}[(i,j)$-$b(A)]$.

Theorem 2. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly pairwise b-irresolute if and only if for any (i,j)-b-open set V in Y, $f^{-1}(V) \subset (i,j)$-$b(f^{-1}[(i,j)$-$b(V)])$, $i \neq j$, i,j=1,2.*

Proof. Suppose that f is weakly pairwise b-irresolute and let V be any (i,j)-b-open set in Y. Then for any $x \in X$ with $x \in f^{-1}(V)$, there exists some (i,j)-b-open set U in X such that $x \in U$ and $f(U) \subset (i,j)$-$b(V)$. Hence $x \in U \subset f^{-1}[(i,j)$-$b(V)]$. Consequently , $x \in (i,j)$-$b(f^{-1}[(i,j)$-$b(V)])$ and $f^{-1}(V) \subset (i,j)$-$b(f^{-1}[(i,j)$-$b(V)])$. To show the converse part, we let $x \in X$ and V be any (i,j)-b-open set in Y with $f(x) \in V$. Then with the given condition $f^{-1}(V) \subset (i,j)$-$b(f^{-1}((i,j)$-$b(V)))$ we let $U = (i,j)$-$b(f^{-1}[(i,j)$-$b(V)])$. Then, (i,j)-b-open subset U is such that $x \in U \subset f^{-1}[(i,j)$-$b(V)]$. Therefore $f(U) \subset (i,j)$-$b(V)$

Theorem 3. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly pairwise b-irresolute if and only if for any subset A of Y, (i,j)-$b(f^{-1}[(i,j)$-$b((i,j)$-$b(A)])) \subset f^{-1}[(i,j)$-$b(A)]$, $i \neq j$, i,j=1,2.*

Proof. Let A be any subset of Y and $x \in X$ be such that $x \notin f^{-1}[(i,j)$-$b(A)]$. Then $f(x) \notin (i,j)$-$b(A)$ and so there exists some (i,j)-b-open set W in Y such that $f(x) \in W$ and $W \cap A = \emptyset$. f being weakly pairwise b-irresolute, *there exists some (i,j)-b-open set U in X such that $x \in U$ and $f(U) \subset (i,j)$-$b(W)$. Further, $W \cap (i,j)$-$b(A) = \emptyset$ and (i,j)-$b[Y \backslash (i,j)$-$b(A)] = [Y \backslash (i,j)$-$b((i,j)$-$b(A)])$. Therefore, $f(U) \subset [Y \backslash (i,j)$-$b((i,j)$-$b(A)])$ and hence, $f(U) \cap (i,j)$-$b((i,j)$-$b(A)) = \emptyset$. Consequently, $U \cap f^{-1}[(i,j)$-$b((i,j)$-$b(A)]) = \emptyset$. It follows that $x \notin (i,j)$-$b(f^{-1}[(i,j)$-$b((i,j)$-$b(A)]))$. Hence (i,j)-$b(f^{-1}[(i,j)$-$b((i,j)$-$b(A)]) \subset f^{-1}[(i,j)$-$b(A)]$. Conversely, let $x \in X$ and V be any (i,j)-b-open set in Y with $f(x) \in V$. Then $V \cap [Y \backslash (i,j)$-$b(V)] = \emptyset$. Therefore $f(x) \notin (i,j)$-$b[Y \backslash (i,j)$-$b(V)]$ and hence $x \notin f^{-1}[(i,j)$-$b[Y \backslash (i,j)$-$b(V)]]$. Now $[Y \backslash (i,j)$-$b(V)] \subset (i,j)$-$b((i,j)$-$b[Y \backslash (i,j)$-$b(V)])$ and by hypothesis, (i,j)-$b(f^{-1}[(i,j)$-$b((i,j)$-$b[Y \backslash (i,j)$-$b(V)]])) \subset f^{-1}[(i,j)$-$b[Y \backslash (i,j)$-$b(V)]]$. Therefore $x \notin (i,j)$-$b(f^{-1}[Y \backslash (i,j)$-$b(V)])$. Therefore there exists some (i,j)-b-open set U in X such that $x \in U$ and $U \cap f^{-1}[Y \backslash (i,j)$-$b(V)] = \emptyset$. Consequently, $U \subset X \backslash f^{-1}[Y \backslash (i,j)$-$b(V)] = f^{-1}[(i,j)$-$b(V)]$. Therefore, it follows tht $f(U) \subset (i,j)$-$b(V)$.*

Theorem 4. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following are equivalent:*

1. *The function f is weakly pairwise b-irresolute.*
2. *For each $A \subset Y$, (i,j)-$b(f^{-1}[(i,j)$-$b(i,j)$-$b(A)]) \subset f^{-1}[(i,j)$-$b(A)]$.*
3. *For each (i,j)-b-open set G in Y, (i,j)-$b(f^{-1}(G)) \subset f^{-1}[(i,j)$-$b(G)]$.*
4. *For each (i,j)-b-closed set H in Y,(i,j)-$b(f^{-1}[(i,j)$-$b(H)]) \subset f^{-1}(H)$.*
5. *For each (i,j)-b-open set G in Y, $f^{-1}(G) \subset (i,j)$-$b(f^{-1}[(i,j)$-$b(G)])$. Here, $i \neq j$, i,j=1,2.*

Proof. The straight forward proof be omitted.

Theorem 5. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Y, \eta_1, \eta_2)$ are weakly pairwise b-irresolute functions, then their composition is also weakly pairwise b-irresolute.*

Proof. Let $x \in X$ and W be any (i,j)-b-open subset of Z such that $(g \circ f)(x) \in W$. Since g is weakly pairwise b-irresolute, there exists an (i,j)-b-open set V in Y containing $f(x)$ such that $V \subset g^{-1}((i,j)$-$b(W))$. Further f being weakly pairwise b-irresolute, there exists an (i,j)-b-open set U in X such that $x \in U \subset f^{-1}((i,j)$-$b(V))$. Thus $x \in U \subset f^{-1}[(i,j)$-$b(g^{-1}(i,j)$-$b(W)])$. But g being weakly pairwise b-irresolute (i,j)-$b(g^{-1}(W)) \subset g^{-1}((i,j)$-$b(W))$. Therefore, $x \in U \subset (g \circ f)^{-1}[(i,j)$-$b(W)]$. Consequently, $g \circ f$ is weakly pairwise b-irresolute.

Theorem 6. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function and $g : X \to X \times Y$ the graph of f given by $g(x) = (x, f(x))$ for $x \in X$. If $g : X \to X \times Y$ is weakly pairwise b-irresolute, then f is weakly pairwise b-irresolute.*

Proof. *Let* $x \in X$ *and* V *be an* (i,j)*-b-open set containing* $f(x)$ *in* V*. Then* $X \times V$ *is* (i,j)*-b-open in* $X \times Y$ *containing* $g(x)$*. Since* g *is weakly pairwise b-irresolute, there exists an* (i,j)*-b-open set* U *containing* x *in* X *such that* $g(U) \subset (i,j)\text{-}b(X \times V) \subset X \times (i,j)\text{-}b(V)$*. Since* g *is the graph of* f*, we have* $f(U) \subset (i,j)\text{-}b(V)$*. This shows that* f *is weakly pairwise b-irresolute.*

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