LINEAR BOUNDARY ELEMENTS FOR SOLVING THE NONSINGULAR BOUNDARY INTEGRAL EQUATION OF THE FLUID FLOW AROUND OBSTACLES

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Abstract. The aim of the paper is to present a numerical solution for the problem of the 2D fluid flow around an obstacle, a solution obtained by applying the boundary element method. It is focused on the second step in applying the mentioned method. In order to satisfy the conditions required by the unknown of the problem, the solution is obtained by using linear boundary elements for solving the boundary integral equation of the problem. The boundary integral equation considered is a nonsingular one, and so the numerical solution is expected to be better than the numerical solution of the same problem solved when a singular boundary integral equation is considered. A computer code in MathCAD, based on the method described, is also made in order to get the numerical solution. This solution is compared with the exact one for some particular cases, and with the one obtained in case when a singular integral equation, equivalent with the same problem, is considered. The graphics show a high agreement between the numerical solution and the exact one. It is also pointed out the biggest advantage of the present solution: it is obtained without considering singular integrals, and special methods to treat them, and so all coefficients that appear are easier to evaluate with a computer code.

Keywords: nonsingular boundary integral equation, linear boundary element, boundary element method, subsonic flow, numerical solution.

Mathematics Subject classification 2000: 74S15

1. Introduction

There are two main techniques which can be applied when solving problems of fluid mechanics by BEM: the direct technique and the indirect technique with sources or vortex distributions. Both of them bring the real advantage of the BEM over other numerical methods, the fact that they reduce the problem dimension by one, but both offer an equivalent model of the problem, in terms of singular boundary integral equations. When solving singular integral equations difficult problem arise when evaluating singular integrals and near singular integrals. Special techniques must be considered in order to overpass this aspect, because an improper evaluation of these integrals brings large errors in the numerical formulation and so they can influence the well behavior of the problem to be solved.

For some problems there can be used other techniques for finding the integral formulation of the problem, which lead to nonsingular integrals, as for example the method of regularization. Starting with such a formulation for the 2D problem of the inviscid fluid flow around an obstacle we want to find a numerical solution based on linear isoparametric boundary elements. We consider that the uniform, steady, 2D potential motion of an ideal inviscid fluid of subsonic velocity $\overline{U}_\infty\overline{i}$, pressure $p_\infty$and density $\rho_\infty$is perturbed by the presence of a fixed body of a known boundary, assumed to be smooth and closed. The objective is to find the perturbed motion, and the fluid action on the body. In [3] a nonsingular boundary integral is deduced. In the herein paper we use linear isoparametric boundary elements to solve the problem. We so reduce, by discretization, the integral equation to an algebraic system and the solution of this system is then used to calculate the perturbation velocity and the pressure coefficient on the body. For understanding its signification, specially for better understanding the unknowns and the variables that appear in it, we present it in the following paragraph.

The boundary integral equation is formulated in velocity vector terms and uses the fundamental solution of source type (can also be found in [3]) , in fact the fundamental solution of the system (we consider dimensionless variables):

$$\begin{aligned} &\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\delta\left(x-x_0,y-y_0\right)\\ &\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=0 \end{aligned} \tag{1}$$

given by the following expressions:

$$u^{*}\left(\overline{x},\overline{x}_0\right)=\frac{1}{2\pi}\frac{x-x_0}{\left(x-x_0\right)^2+\left(y-y_0\right)^2}\ ,\ v^{*}\left(\overline{x},\overline{x}^0\right)=\frac{1}{2\pi}\frac{y-y_0}{\left(x-x_0\right)^2+\left(y-y_0\right)^2},\tag{2}$$

$$\overline{v}^{*}\left(\overline{x},\overline{x}^0\right)=\frac{1}{2\pi}\frac{\overline{x}-\overline{x}_0}{\left(x-x_0\right)^2+\left(y-y_0\right)^2}\tag{3}$$

First, it is deduced an integral formulation for the perturbation velocity, $\overline{v}\left(\overline{x}_0\right)$, which stands for $\overline{x}_0$ in the fluid domain, but also for $\overline{x}_0\in C$:

$$\overline{v}\left(\overline{x}_0\right)=\int\limits_C\left\{\left[\overline{n}\left(\overline{v}-\overline{v}_0\right)\right]\overline{v}^{*}+\left[\overline{n}\times\left(\overline{v}-\overline{v}_0\right)\right]\times\overline{v}^{*}\right\}ds\tag{4}$$

where $\overline{v}^{*}$ is the fundamental solution given by (2), and $\overline{n}$ is the unit normal vector at C, inward the fluid

The above boundary integral is a nonsingular one because $\lim\limits_{\overline{x}\to\overline{x}_0\in C}\left(\overline{v}-\overline{v}_0\right)=0$.

On C, the following relations hold: $\overline{n}\times\overline{v}=v_s\overline{k}$, and $\left[\overline{n}\left(\overline{v}-\overline{v}_0\right)\right]\overline{v}^{*}=-n_x\overline{v}^{*}-n_x^0\left(\overline{n}^0\cdot\overline{n}\right)\overline{v}^{*}-v_s^0\left(\overline{n}\cdot\overline{s}^0\right)\overline{v}^{*}$, where $\overline{k}$ is the versor of Oz, v_s is the tangential component of the velocity, $\overline{n}=n_x\overline{i}+n_y\overline{j}$ is the unit normal vector at C, inward the fluid, $\overline{n}^0=\overline{n}\left(\overline{x}_0\right)=n_x^0\overline{i}+n_y^0\overline{j}$ and $\overline{s}^0$ is the unit tangential vector at C evaluated in $\overline{x}_0$.

Using some elementary formulas and some consideration about the components of the unit normal vector at C, and the unit tangential vector, further there is obtained the following boundary integral representation, formulated in velocity vector terms too:

$$\begin{aligned}\overline{v}\left(\overline{x}_0\right)\ &=\ \int\limits_C\left\{v_s\overline{k}\times\overline{v}^{*}+v_s^0\left[\left(\overline{v}^{*}\cdot\overline{s}^0\right)\overline{n}-\left(\overline{n}\cdot\overline{v}^{*}\right)\overline{s}^0-\left(\overline{n}\cdot\overline{s}^0\right)\overline{v}^{*}\right]-n_x\overline{v}^{*}\right\}ds-\\&-\int\limits_C n_x^0\left[\left(\overline{n}^0\cdot\overline{v}^{*}\right)\overline{n}-\left(\overline{n}\cdot\overline{v}^{*}\right)\overline{n}^0-\left(\overline{n}\cdot\overline{n}^0\right)\overline{v}^{*}\right]ds\end{aligned}\tag{5}$$

where there are used the same notations as before.

After doing the vectorial product $\overline{n}^0\times\overline{v}\left(\overline{x}_0\right)$, there is deduced the boundary integral representation for the tangential component of the velocity. The tangential component of the perturbation velocity in any point of the boundary,

$\overline{x}_0 \in C$, has the following expression:

$$v_s^0 = \int_C \left\{ \left[\left(\overline{n}^0 \cdot \overline{v}^* \right) v_s - \left(\overline{n}^0 \cdot \overline{v}^* \right) v_s^0 \right] + \left[\left(\overline{v}^* \cdot \overline{s}^0 \right) n_x^0 - \left(\overline{v}^* \cdot \overline{s}^0 \right) n_x \right] \right\} ds \qquad (6)$$

Equation (6) is the nonsingular integral equation of the problem (for more information about obtaining this boundary integral equation see [3]).

In order to solve the integral equation we consider a boundary mesh using linear boundary elements. For a particular case, when the problem has an analytical solution, we make a comparison between the numerical solutions obtained and the exact one in order to validate the method proposed.

2. Solving the nonsingular integral equation by using linear isoparametric boundary elements

In the boundary element approach used herein, for solving the integral equation (7), we consider linear isoparametric boundary elements, so the case when the geometry and the unknown are local approximated by linear models that use the same base functions. The boundary C is divided into N linear segments, noted $L_j, j = \overline{1, N}$, with ends in , $\overline{x}_j, \overline{x}_{j+1}, j = \overline{1, N}$, $\overline{x}_{N+1} = \overline{x}_1$; the extremes of the segments being situated on C. So we approximate the contour C with a polygonal line. The extremes of the segment L_j are noted x_j^1, x_j^2,in a local numbering. We have relations: $\overline{x}_j^2 = \overline{x}_{j+1}^1, \forall i \in \{1, 2, ..., N-1\}$,and $\overline{x}_N^2 = \overline{x}_1^1$, contour C being closed.

For evaluating the integrals we use a local system of coordinates (see [1], [2]) with the origin in the first node of an element , and so we have, for a boundary element, the relations:

$$\overline{x} = \overline{x}_j^1 \varphi_1 + \overline{x}_j^2 \varphi_2 \qquad (7)$$

where φ_1, φ_2 are the shape functions given by: $\varphi_1 = 1 - t, \varphi_2 = t, t \in [0, 1]$.

Using isoparametric boundary elements (see [1], [2]) we have, for the unknown function v_s , noted w for simplifying the writing, the local representation:

$$w = w_j^1 \varphi_1 + w_j^2 \varphi_2$$

where w_j^1, w_j^2 are the nodal values of the unknown, it means the values of w at the extremes of the boundary element L_j, in the local numbering. Considering first the case of a smooth boundary these values satisfy the relations: $w_j^2 = w_{j+1}^1, \forall i \in \{1, 2, ..., N-1\}$, and $w_N^2 = w_1^1$.

Equation (6) becomes:

$$w^0 = \sum_{j=1}^{N} \int_{L_j} \left\{ \left[\left(\overline{n}^0 \cdot \overline{v}^* \right) w - \left(\overline{n}^0 \cdot \overline{v}^* \right) w^0 \right] + \left[\left(\overline{v}^* \cdot \overline{s}^0 \right) n_x^0 - \left(\overline{v}^* \cdot \overline{s}^0 \right) n_x \right] \right\} ds \tag{8}$$

For $\overline{x}_0 = \overline{x}_i^1$, it becomes:

$$\begin{aligned} w_i^1 &= \sum_{j=1}^{N} \int_{L_j} \left[\left(\overline{n}^i \cdot \overline{v}^* \left(\overline{x}, \overline{x}_i^1 \right) \right) w - \left(\overline{n}^i \cdot \overline{v}^* \left(\overline{x}, \overline{x}_i^1 \right) \right) w_i^1 \right] ds + \\ &\quad + \sum_{j=1}^{N} \int_{L_j} \left[\left(\overline{v}^* \left(\overline{x}, \overline{x}_i^1 \right) \cdot \overline{s}^i \right) n_x^i - \left(\overline{v}^* \left(\overline{x}, \overline{x}_i^1 \right) \cdot \overline{s}^i \right) n_x \right] ds \end{aligned} \tag{9}$$

$$w_i^1 = \sum_{j=1}^{N} \int_{L_j} \left\{ \left(\overline{n}^i \cdot \overline{v}^* \left(\overline{x}, \overline{x}_i^1 \right) \right) \left(w - w_i^1 \right) + \left(\overline{v}^* \left(\overline{x}, \overline{x}_i^1 \right) \cdot \overline{s}^i \right) \left(n_x^i - n_x \right) \right\} ds \tag{10}$$

Using relation $ds = l_j dt$, and introducing the local behavior of the unknown function on L_j,we obtain:

$$\begin{aligned} w_i^1 &= \sum_{j=1}^{N} l_j \int_0^1 \left(n_x^i \cdot u^* \left(\overline{x}, \overline{x}_i^1 \right) + n_y^i \cdot v^* \left(\overline{x}, \overline{x}_i^1 \right) \right) \left(w_j^1 \varphi_1 + w_j^2 \varphi_2 - w_i^1 \right) dt + \\ &= \sum_{j=1}^{N} l_j \int_0^1 \left(\overline{v}^* \left(\overline{x}, \overline{x}_i^1 \right) \cdot \overline{s}^i \right) \left(n_x^i - n_x \right) dt \end{aligned} \tag{11}$$

Taking into account the expressions of the fundamental solutions (3) and the relation between $\overline{n}$ and $\overline{s}$, $\overline{s} = -n_y \overline{i} + n_x \overline{j}$, we get:

$$\begin{aligned} w_i^1 = \sum_{j=1}^{N} \frac{l_j}{2\pi} \int_0^1 \left(n_x^i \frac{x - x_i^1}{\left(x - x_i^1\right)^2 + \left(y - y_i^1\right)^2} + n_y^i \frac{y - y_i^1}{\left(x - x_i^1\right)^2 + \left(y - y_i^1\right)^2} \right) \\ \left(w_j^1 \varphi_1 + w_j^2 \varphi_2 - w_i^1 \right) dt + \end{aligned}$$

$$+\sum_{j=1}^{N}\frac{l_j}{2\pi}\int_0^1\left(-n_y^i\frac{x-x_i^1}{\left(x-x_i^1\right)^2+\left(y-y_i^1\right)^2}+n_x^i\frac{y-y_i^1}{\left(x-x_i^1\right)^2+\left(y-y_i^1\right)^2}\right)\left(n_x^i-n_x\right)dt \tag{12}$$

where $l_j=\left(\left(x_j^2-x_j^1\right)^2+\left(y_j^2-y_j^1\right)^2\right)^{\frac{1}{2}}=\left\|\overline{x}_j^2-\overline{x}_j^1\right\|$.

Further we introduce in the above equation the geometry of the boundary element: $x=x_j^1\varphi_1+x_j^2\varphi_2$, $y=y_j^1\varphi_1+y_j^2\varphi_2$.

$$w_i^1=\sum_{j=1}^{N}\frac{l_j}{2\pi}\int_0^1\left(n_x^i\frac{\left(x_j^2-x_j^1\right)t+x_j^1-x_i^1}{a_jt^2+2b_{ij}t+c_{ij}}+n_y^i\frac{\left(y_j^2-y_j^1\right)t+y_j^1-y_i^1}{a_jt^2+2b_{ij}t+c_{ij}}\right)$$
$$\left(\left(w_j^2-w_j^1\right)t+w_j^1-w_i^1\right)dt+$$

$$+\sum_{j=1}^{N}\frac{l_j}{2\pi}\int_0^1\left(-n_y^i\frac{\left(x_j^2-x_j^1\right)t+x_j^1-x_i^1}{a_jt^2+2b_{ij}t+c_{ij}}+n_x^i\frac{\left(y_j^2-y_j^1\right)t+y_j^1-y_i^1}{a_jt^2+2b_{ij}t+c_{ij}}\right)\left(n_x^i-n_x\right)dt \tag{13}$$

We have used the following notations:

$$\begin{aligned}
a_j &= l_j^2, \\
b_{ij} &= \left(x_j^1-x_i^1\right)\left(x_j^2-x_j^1\right)+\left(y_j^1-y_i^1\right)\left(y_j^2-y_j^1\right), \\
c_{ij} &= \left(x_j^1-x_i^1\right)^2+\left(y_j^1-y_i^1\right)^2.
\end{aligned} \tag{14}$$

Further we get:

$$\begin{aligned}
w_i^1 &= \sum_{j=1}^{N}\tfrac{l_j}{2\pi}\left(w_j^2-w_j^1\right)\int_0^1\left(\tfrac{n_x^i\left[\left(x_j^2-x_j^1\right)t+x_j^1-x_i^1\right]+n_y^i\left[\left(y_j^2-y_j^1\right)t+y_j^1-y_i^1\right]}{a_jt^2+b_{ij}t+c_{ij}}t\right)dt+ \\
&\quad+\sum_{j=1}^{N}\tfrac{l_j}{2\pi}\left(w_j^1-w_i^1\right)\int_0^1\left(\tfrac{n_x^i\left[\left(x_j^2-x_j^1\right)t+x_j^1-x_i^1\right]+n_y^i\left[\left(y_j^2-y_j^1\right)t+y_j^1-y_i^1\right]}{a_jt^2+b_{ij}t+c_{ij}}\right)dt+
\end{aligned}$$

$$+\sum_{j=1}^{N}\frac{l_j}{2\pi}\int_0^1\left(-n_y^i\frac{\left(x_j^2-x_j^1\right)t+x_j^1-x_i^1}{a_jt^2+b_{ij}t+c_{ij}}+n_x^i\frac{\left(y_j^2-y_j^1\right)t+y_j^1-y_i^1}{a_jt^2+b_{ij}t+c_{ij}}\right)\left(n_x^i-n_x\right)dt \tag{(15)}$$

Taking into account the going sense on C we have the following relations for the components of the unit normal vector in $\overline{x}_j$: $n_x^j = \frac{y_j^2 - y_j^1}{l_j}$, $n_y^j = \frac{x_j^1 - x_j^2}{l_j}, \forall j = \overline{1, N}$. In case of a linear boundary element we notice that everywhere on L_j $n_x = n_x^j, \quad n_y = n_y^j$.With the notations:

$$I_k^{ij} = \int_0^1 \frac{t^k dt}{a_j t^2 + b_{ij} t + c_{ij}}, \quad k = \overline{0, 2} \tag{15}$$

$$P_{ij} = \frac{l_j}{2\pi} \left\{ \left[n_x^i \left(x_j^2 - x_j^1 \right) + n_y^i \left(y_j^2 - y_j^1 \right) \right] I_2^{ij} + \left[n_x^i \left(x_j^1 - x_i^1 \right) + n_y^i \left(y_j^1 - y_i^1 \right) \right] I_1^{ij} \right\} \tag{16}$$

$$R_{ij} = \frac{l_j}{2\pi} \left\{ \left[n_x^i \left(x_j^2 - x_j^1 \right) + n_y^i \left(y_j^2 - y_j^1 \right) \right] I_1^{ij} + \left[n_x^i \left(x_j^1 - x_i^1 \right) + n_y^i \left(y_j^1 - y_i^1 \right) \right] I_0^{ij} \right\} \tag{17}$$

$$\begin{aligned} S_{ij} &= \frac{l_j \left(n_x^i - n_x^j \right)}{2\pi} \left[-n_y^i \left(x_j^2 - x_j^1 \right) + n_x^i \left(y_j^2 - y_j^1 \right) \right] I_1^{ij} + \\ &\quad + \frac{l_j \left(n_x^i - n_x^j \right)}{2\pi} \left[-n_y^i \left(x_j^1 - x_i \right) + n_x^i \left(y_j^1 - y_i \right) \right] I_0^{ij} \end{aligned} \tag{18}$$

we get the following equivalent form for equation (15):

$$w_i^1 = \sum_{j=1}^{N} P_{ij} \left(w_j^2 - w_j^1 \right) + \sum_{j=1}^{N} R_{ij} \left(w_j^1 - w_i^1 \right) + \sum_{j=1}^{N} S_{ij} \tag{19}$$

Because i can take all values between 1 and N we have reduced the problem at a system of N linear equations of the above form.

3. Numerical evaluation of the coefficients

As it can be seen all the coefficients that appear depend on the coordinates of the nodes chosen for the boundary discretization and can be exactly evaluated. Obvious they are usually integrals when $\overline{x}_i^1$ is not one of L_j extremes. In papers [5], [6] the same problem is solved using linear boundary elements too, but in those cases the boundary equivalent representations of the problem

were, in all cases, singular boundary integral equations. When solving such boundary integral equations it is quite difficult to handle, and to evaluate, the singular integrals, and the near singular integrals that appear, see [7] for more details about singular integrals. Special techniques, some of them quite difficult to apply, must be used in such cases for obtaining a good accuracy for the numerical solutions. What is interesting in this approach, is the fact that we don't have to overpass such problems, because the boundary integral equation is not a singular one. As we shall see we can obtain analytical expression even in the case when $\overline{x}_i^1$ is one of L_j extremes.
For $\overline{x}_i^1 \neq \overline{x}_j^1$,in fact for $i \neq j$, the analytical expressions for these coefficients can be easily obtained:

$$\begin{aligned} I_0^{ij} &= \frac{1}{\sqrt{a_j c_{ij} - b_{ij}^2}} \arctan \frac{\sqrt{a_j c_{ij} - b_{ij}^2}}{c_{ij} + b_{ij}}, I_1^{ij} = \frac{1}{2a_j} \ln \frac{a_j + 2b_{ij} + c_{ij}}{c_{ij}} - \\ & - \frac{b_{ij}}{a_j \sqrt{a_j c_{ij} - b_{ij}^2}} \arctan \frac{\sqrt{a_j c_{ij} - b_{ij}^2}}{c_{ij} + b_{ij}}, \qquad (20) \\ I_2^{ij} &= \frac{1}{a_j} - \frac{b_{ij}}{a_j^2} \ln \frac{a_j + 2b_{ij} + c_{ij}}{c_{ij}} + \frac{2b_{ij}^2 - a_j c_{ij}}{a_j^2} I_0^{ij}. \qquad (21) \end{aligned}$$

Considering now that $\overline{x}_i^1 = \overline{x}_j^1$, so that $i = j$, we first notice that the corresponding term of the middle sum from the right hand site vanishes, so we can consider that $R_{ii} = 0$. Using the same notations as before we deduced that $b_{ij} = 0$, $c_{ij} = 0$, and so we have:

$$\begin{aligned} I_2^{ii} &= \frac{1}{a_j}, \qquad S_{ii} = 0 \qquad (22) \\ P_{ii} &= \frac{1}{2\pi l_j} \left[n_x^i \left(x_i^2 - x_i^1 \right) + n_y^i \left(y_i^2 - y_i^1 \right) \right] \qquad (23) \end{aligned}$$

All the coefficients that appear in equation (20) depend only on the nodes chosen for the boundary discretization. Returning to the global system of notation, that mean considering $w(\overline{x}_i^1) = w_i,\ i = \overline{1, N},$ we have $w_j = w_j^1 = w_{j-1}^2, j = \overline{2, N}$, $w_1 = w_1^1 = w_N^2$. Conveniently grouping the terms in (20), we finally obtain the following equivalent system, in terms of nodal unknowns, the nodal values of the tangential component of the velocity on the boundary:

$$\sum_{j=1}^{N} A_{ij} w_j = B_i \ , \ i = \overline{1,N} \tag{24}$$

where

$$\begin{aligned} A_{ij} &= -R_{ij} + P_{ij} - P_{ij-1}, \ i = \overline{2,N}, i \neq j, \ A_{i1} = -R_{i1} + P_{i1} - P_{iN} \ , i \neq 1 \\ A_{ii} &= 1 + \sum_{j=1}^{N} R_{ij} + P_{ii} - P_{ii-1}, \ i = \overline{2,N}, \ A_{11} = 1 + \sum_{j=1}^{N} R_{1j} + P_{11} - P_{iN} \\ B_i &= \sum_{j=1}^{N} S_{ij}, \ i = \overline{1,N} \end{aligned} \tag{25}$$

So all the coefficients in system (25) can be computed analytically, so, notwithstanding the truncation errors that arise, they can be exactly evaluated with a computer code. After solving system (25) we obtain the numerical results for the nodal values of the perturbation velocity.

4. Numerical results and conclusions

In order to test the method, we consider the case of the circular obstacle. In this particular case the problem has an exact solution (see [3], [4]). The exact solution furnishes the following expressions for the components of the velocity in a point $M(R,\theta)$ situated on the circle $C(O,R)$ (the quantities we use are also dimensionless): $u = -\cos 2\theta$, $v = -\sin 2\theta$. These components are used to evaluate the local pressure coefficient, but also the tangential component of the velocity. So, after the components of the velocity are found, the exact values for the tangential velocity, W, can be calculate with the formula:

$$W = n_x v - n_y u \tag{26}$$

With a computer code made in MathCAD we evaluate, based on the method exposed, the numerical values for the nodal tangential velocities on C, and the exact ones. The results are shown in Fig.1.As it can be seen we have used 20 nodes for the boundary discretization. The comparison between the exact values and the numerical ones shows a high degree of accuracy, even for a small number of nodes on the boundary. Better results can be obtained when using more nodes for the boundary discretization.

In paper [5] , considering the indirect boundary element method with sources distribution, which implies a singular boundary integral equation for the problem (SBIE), and solving it with linear isoparametric boundary elements too, it is obtained a numerical solution for the same particular case, the circular obstacle. In order to establish which method offers better results we compare, in Fig.2., the numerical solutions obtained with the exact one. As we can see, the calculated and the analytical values of the tangential velocity are very close , in both cases, but better results are obtained with the present method. This is an expected result because the numerical integration doesn't imply numerical evaluations of singular integrals and so it improves the numerical solution accuracy. The present method has another big advantage too, namely that it is easier to apply because it deals only with nonsingular integrals which doesn't need special treatments. When the boundary integral formulation of the problem implies singular integrals it is difficult to carry out their numerical evaluation. An improper method to evaluate them can affect the accuracy of the numerical solution. Finding good techniques for the singular integrals numerical evaluation represents a great challenge and the most difficult step to overpass.

We have considered in this paper only the case of a smooth obstacle, but taking into account a Kutta-Jukovsky condition, it can be applied to obstacles with cusped trailing edge too. It is also interesting to try to extend the method to the compressible case too, if it is possible.

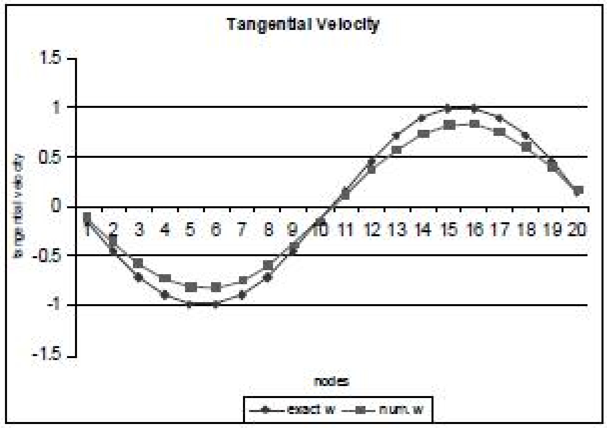

Fig. 1. The nodal values of the tangential velocity - exact and numerical solution; circular obstacle; 20 nodes.

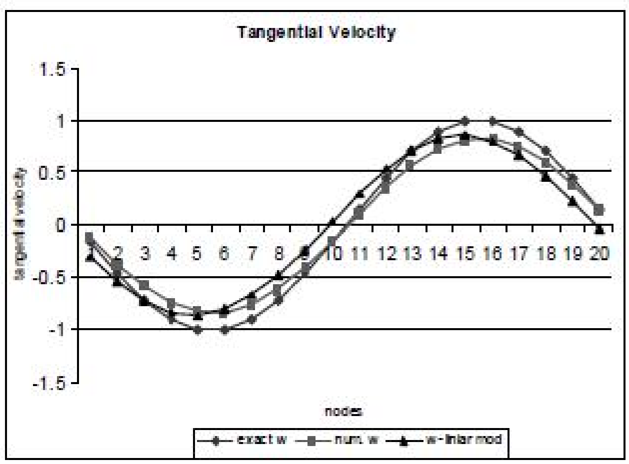

Fig. 2. The nodal values of the tangential velocity - exact, numerical solution, numerical solution in case of SBIE; circular obstacle; 20 nodes.

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