Acta Universitatis Apulensis No. 38/2014
ISSN: 1582-5329 pp. 163-176

THE SHOCK PROFILE WAVE PROPAGATION OF KURAMOTO-SIVASHINSKY EQUATION AND SOLITONIC SOLUTIONS OF GENERALIZED KURAMOTO-SIVASHINSKY EQUATION

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Abstract. The Application of Kuramoto-Sivashinsky (KS) equation is in complicated fluid dynamics systems. Its chaotic pattern forming behavior is important. In this paper this equation and generalized Kuramoto-Sivashinsky equation (GKS) are solved by sinc-collocation method. A mesh free technique is applied to solve this equation.

2000 *Mathematics Subject Classification*: 76L05; 49Mxx.

Keywords: numerical method, sinc, collocation.

1. Introduction

The appearance of shock wave or strong pressure wave is in elastic medium like air, water or a solid substance that is produced by supersonic aircraft, explosions, lightning or other cases that produces violent variation in pressure. The difference between sock waves and sound wave is the propagations of shock waves. The speeds of shock waves depends on amplitude. The amplitude of a strong shock decreases almost as the inverse square of the distance until the wave has become so weak that it relates the laws of acoustic waves. The application of shock waves is the study the equation of state of any material.

A soliton is a special kind of solitary wave, which is not destroyed when it collides with another wave of the same kind. The first appearance of soliton was about 2 centuries ago in August of 1834 near Edinburgh, Scotland. Scottish scientist and engineer John Scott Russell (1808- 1882) first observed the soliton, translation wave or the great solitary wave. Russell spent many years to study about this phenomenon. The main role of solitons are studied in physics, mathematics, hydromechanics, astrophysics, meteorology, oceanography and biology. Among many-faced solitons, there are three most famous solitons: the KdV solitons (Russells solitons), the FK (Sine- Gordon) solitons and the envelope (group) solitons. The application of Russell-KdV solitons is in physical systems with weakly nonlinear and weakly dispersive waves.

Consider Kuramoto-Sivashinsky (KS) equation as

$$u_t + uu_x + \alpha u_{xx} + u_{xxxx} = 0, \tag{1}$$

with the initial condition

$$u(x,0) = f(x), \tag{2}$$

and the boundary conditions

$$u(a,t) = g_a(t), \;\; u(b,t) = g_b(t), t \geq 0, \tag{3}$$

where α is a constant. KS equation [1] is a canonical evolution equation which has attracted considerable importance in last years. The linear terms describe a balance between long-wave instability and short-wave stability, with the nonlinear term providing a mechanism foe energy transfer between wave modes. It arises in a broad spectrum of contexts and admits various fascinating solutions like traveling waves of permanent form and chaos. It is one of the simplest partial differential equations which is capable of exhibiting chaotic behavior. The chaotic behavior typically occurs when the equation is integrated over finite $x-$domain with periodic boundary conditions. It has various applications, e.g., long waves on thin films, long waves on the interface between two viscous fluids, unstable drift waves in plasmas, reaction diffusion systems and flame front instability. It also describes the fluctuations of the position of a flame front, the motion of a fluid going down a vertical wall, or a spatially uniform oscillating chemical reaction in a homogeneous medium.

Consider generalized Kuramoto-Sivashinsky (GKS) equation as

$$u_t + uu_x + u_{xx} + \sigma u_{xxx} + u_{xxxx} = 0, \tag{4}$$

with the initial condition

$$u(x,0) = f(x), \tag{5}$$

and the boundary conditions

$$u(a,t) = g_a(t), \;\; u(b,t) = g_b(t), t \geq 0, \tag{6}$$

GKS equation is an important mathematical model arising in many different physical contexts to describe many phenomena which are simultaneously involved in nonlinearity, dissipation, dispersion and instability, especially at the description of turbulence processes.

Kuamoto-Sivashinsky equation has been studied by many authors. Y. Zhang et al. [2] made bifurcation analysis by using the center manifold reduction method, together with the eigenvalue analysis for the Kuramoto-Sivashinsky equation. S. Dubljevic [3] has focused on the model predictive control design methodology that successfully accounts for the state and input constraints applied in the context of highly dissipative Kuramoto-Sivashinsky (KS) partial differential equation (PDE) describing stability of a thin film thickness in the two-phase annular flow in vertical pipes. Y. Bozhkov et al. [4] have conisedered group classification and conservation laws for a two-dimensional generalized Kuramoto-Sivashinsky equation. L. Bo [5] have established a large deviation principle for the (weak) solution to a nonlocal Kuramoto-Sivashinsky stochastic partial differential equation with small noise perturbation. B. Barker et al. [6] have announced a general result resolving the long-standing question of nonlinear modulational stability or stability with respect to localized perturbations of periodic traveling-wave solutions of the generalized Kuramoto-Sivashinsky equation, establishing that spectral modulational stability, defined in the standard way, implies nonlinear modulational stability with sharp rates of decay. D. Yang [7] has investigated the relation between the Kolmogorov operator associated to a stochastic Kuramoto-Sivashinsky equation and the infinitesimal generator for the corresponding transition semigroup.

2. The Sinc function

In this section the basis of sinc function is discussed [8]. The sinc function is defined on the whole real line, $-\infty < x < \infty$, by

$$\operatorname{sinc}(x) = \begin{cases} \frac{\sin(\pi x)}{\pi x}, & x \neq 0; \\ 1, & x = 0. \end{cases} \tag{7}$$

For any $h > 0$, the translated sinc functions with evenly spaced nodes are given as

$$S(j,h)(z) = \operatorname{sinc}(\frac{z - jh}{h}), \quad j = 0, \pm 1, \pm 2, \cdots . \tag{8}$$

The sinc functions are cardinal for the interpolating points $z_k = kh$ in the sense that

$$S(j,h)(kh) = \delta_{jk}^{(0)} = \begin{cases} 1, & k = j; \\ 0, & k \neq j. \end{cases} \tag{9}$$

If f is defined on the real line, then for $h > 0$ the series

$$C(f,h)(z) = \sum_{j=-\infty}^{\infty} f(jh)\operatorname{sinc}(\frac{z - jh}{h}), \tag{10}$$

is called the Whittaker cardinal expansion of f whenever this series converges. They are based in the infinite strip D_s in the complex plane

$$D_s = \{w = u + iv : |v| < d \leq \frac{\pi}{2}\}. \tag{11}$$

Some derivatives of sinc function will be used in reduction the equation to matrix form so,

$$I_{ji}^{(0)} = [S(j,h)(x)]\big|_{x=x_i} = \begin{cases} 1, & j = i; \\ 0, & j \neq i, \end{cases} \tag{12}$$

$$I_{ji}^{(1)} = \frac{d}{dx}[S(j,h)(x)]\big|_{x=x_i} = \frac{1}{h}\begin{cases} 0, & j = i; \\ \frac{(-1)^{(i-j)}}{(i-j)}, & j \neq i, \end{cases} \tag{13}$$

and

$$I_{ji}^{(2)} = \frac{d^2}{dx^2}[S(j,h)(x)]\big|_{x=x_i} = \frac{1}{h^2}\begin{cases} \frac{-\pi^2}{3}, & j = i; \\ \frac{-2(-1)^{(i-j)}}{(i-j)^2}, & j \neq i. \end{cases} \tag{14}$$

$$I_{ji}^{(3)} = \frac{d^3}{dx^3}[S(j,h)(x)]\big|_{x=x_i} = \frac{1}{h^3}\begin{cases} 0, & j = i; \\ \frac{(-1)^{(i-j)}}{(i-j)^3}[6 - \pi^2(i-j)^2], & j \neq i, \end{cases} \tag{15}$$

$$I_{ji}^{(4)} = \frac{d^4}{dx^4}[S(j,h)(x)]\big|_{x=x_i} = \frac{1}{h^4}\begin{cases} \left(\frac{\pi^4}{5}\right), & j = i; \\ \frac{(-1)^{i-j}}{(i-j)^4}(-4 + \frac{2}{3}\pi^2(i-j)^2) & j \neq i, \end{cases} \tag{16}$$

And so on, for even coefficients, where $r = 1, 2, \ldots$

$$\begin{aligned} I_{ji}^{(2r)} &= \frac{d^{2r}}{dx^{2r}}[S(j,h)(x)]\big|_{x=x_i} \\ &= \begin{cases} \left(\frac{\pi}{h}\right)^{2r}\frac{(-1)^r}{2r+1}, & j = i; \\ \frac{(-1)^{(i-j)}}{h^{2r}(i-j)^{2r}}\sum_{l=0}^{r-1}(-1)^{l+1}\frac{2r!}{(2l+1)!}\pi^{2l}(i-j)^{2l}, & j \neq i, \end{cases} \end{aligned} \tag{17}$$

and for odd coefficients, where $r = 1, 2, \ldots$

$$I_{ji}^{(2r+1)} = \frac{d^{2r+1}}{dx^{2r+1}}[S(j,h)(x)]\big|_{x=x_i}$$

$$= \begin{cases} 0 & j=i; \\ \frac{(-1)^{(j-i)}}{h^{2r+1}(j-i)^{2r+1}} \sum_{l=0}^{r}(-1)^l \frac{(2r+1)!}{(2l+1)!}\pi^{2l}(i-j)^{2l}, & j \neq i. \end{cases} \tag{18}$$

3. Structure of the method

Consider forth order partial differential equations Kuramoto-Sivashinsky (KS) equation as

$$u_t + uu_x + \alpha u_{xx} + u_{xxxx} = 0, \tag{19}$$

with the initial condition

$$u(x,0) = f(x), \tag{20}$$

and the boundary conditions

$$u(a,t) = g_a(t), \;\; u(b,t) = g_b(t), t \geq 0, \tag{21}$$

where α is a constant and consider forth order partial differential equation of generalized Kuramoto-Sivashinsky (GKS) equation as

$$u_t + uu_x + u_{xx} + \sigma u_{xxx} + u_{xxxx} = 0, \tag{22}$$

with the initial conidition

$$u(x,0) = f(x), \tag{23}$$

and the boundary conditions

$$u(a,t) = g_a(t), \;\; u(b,t) = g_b(t), t \geq 0. \tag{24}$$

By discretizing time derivative of KS's equation using a classic finite difference formula and space derivatives by θ- weighted scheme we have

$$\begin{aligned} &\frac{u^{n+1}-u^n}{\delta t} + \theta((uu_x)^{n+1} + (\alpha u_{xx})^{n+1} + (u_{xxxx})^{n+1}) \\ &+(1-\theta)((uu_x)^n + (\alpha u_{xx})^n + (u_{xxxx})^n) = 0, \end{aligned} \tag{25}$$

so using Taylor expansion for the term uu_x and considering we have

$$\begin{aligned} &u^{n+1} + \delta t\theta([u^n u_x^{n+1} + u_x^n u^{n+1}] + u_{xx}^{n+1} + u_{xxxx}^{n+1}) \\ &= u^n + ((2\theta-1)(uu_x)^n + \delta t(1-\theta)(u_{xx}^n + u_{xxxx}^n), \end{aligned} \tag{26}$$

for KS equation and with same calculation for GKS equation we have

$$\begin{aligned} &u^{n+1} + \delta t\theta([u^n u_x^{n+1} + u_x^n u^{n+1}] + u_{xx}^{n+1} + \sigma u_{xxx}^{n+1} + u_{xxxx}^{n+1}) \\ &= u^n + \delta t((2\theta - 1)(uu_x)^n - (1-\theta)(u_{xx}^n + \sigma u_{xxx}^{n+1} + u_{xxxx}^n). \end{aligned} \tag{27}$$

Now we use approximate solution as

$$u(x, t^n) = u^n(x) \simeq \sum_{j=1}^{N} u_j^n S_j(x). \tag{28}$$

where

$$S_j(x) = Sin(\frac{x - (j-1)h - a}{h}). \tag{29}$$

By substituting above approximate solution in Eq. (26) a matrix representation is obtained for KS equation as

$$Mu^{n+1} = R, \tag{30}$$

where

$$\begin{aligned} &A_d = [I_{ij}^{(0)} : i = 2, ..., N-1, j = 1, ..., N,\ 0\ elsewhere]_{N\times N}, \\ &A_b = [I_{ij}^{(0)} : i = 1, N, j = 1, \dots, N,\ 0\ elsewhere]_{N\times N}, \\ &B = [I_{ij}^{(1)} : i = 2, \dots, N-1, j = 1, \dots, N,\ 0\ elsewhere]_{N\times N}, \\ &C = [I_{ij}^{(2)} : i = 2, \dots, N-1, j = 1, \dots, N,\ 0\ elsewhere]_{N\times N}, \\ &G = [I_{ij}^{(3)} : i = 2, \dots, N-1, j = 1, \dots, N,\ 0\ elsewhere]_{N\times N}, \\ &H = [I_{ij}^{(4)} : i = 2, \dots, N-1, j = 1, \dots, N,\ 0\ elsewhere]_{N\times N}, \\ &u_x^n = Bu^n, \quad D = u_x^n * A_d, \quad E = (u^n) * B, \\ &F^{n+1} = [g_a(t^{n+1}), 0, \dots, 0, g_b(t^{n+1})]^T, \end{aligned}$$

so with these definition for KS equation we have

$$\begin{aligned} &M = [A_d + A_b + \theta\delta t(E + D + C + H)], \\ &R = [A_d + \delta t\{(2\theta - 1)E - (1-\theta)(C + H)\}]u^n + F^{n+1}, \end{aligned}$$

and with same substitution for GKS's equation

$$\begin{aligned} &M = [A_d + A_b + \theta\delta t(E + D - C + \sigma G + H)], \\ &R = [A_d + \delta t\{(2\theta - 1)E - (1-\theta)(-C + H + \sigma G)\}]u^n + F^{n+1}. \end{aligned}$$

4. Stability Analysis

In this section stability analysis of approximate solution for linearized equation is discussed. The error at nth time level is

$$e^n = u^n_{exact} - u^n_{approximate}.$$

4.1. KS's equation

By considering the obtained matrix we have

$$[H + \delta t\theta K]e^{n+1} = [H - \delta t(1-\theta)K]e^n, \tag{31}$$

where $H = [A_d + A_b]A^{-1}$ and $K = [E + D - C + H]A^{-1}$ so,

$$e^{n+1} = Pe^n,$$

where $P = [H + \delta t\theta K]^{-1}[H - \delta t(1-\theta)K]$. This method is stable [9] if $\parallel P \parallel_2 \leq 1$ or $\rho(P) \leq 1$ which is spectral radius of the matrix P. The stability is assured if all the eigenvalues of the matrix $[H + \delta t\theta K]^{-1}[H - \delta t(1-\theta)K]$ satisfy the following condition

$$\left|\frac{\lambda_H - \delta t(1-\theta)\lambda_K}{\lambda_H + \delta t\theta\lambda_K}\right| \leq 1, \tag{32}$$

where λ_H and $\lambda_K$are eigenvalues of the matrices H and K respectively. When $\theta = 0.5$, the inequality (32) becomes

$$\left|\frac{\lambda_H - 0.5\delta t\lambda_K}{\lambda_H + 0.5\delta t\lambda_K}\right| \leq 1. \tag{33}$$

In the case of complex eigenvalues $\lambda_H = a_h + ib_h$ and $\lambda_K = a_k + ib_k$, where a_h, a_k, b_h and b_k are any real numbers, the inequality (33) takes the following form,

$$\left|\frac{(a_h - 0.5\delta t a_k) + i(b_h - 0.5\delta t b_k)}{(a_h - 0.5\delta t a_k) + i(b_h - 0.5\delta t b_k)}\right| \leq 1. \tag{34}$$

The inequality (34) is satisfied if $a_h a_k + b_h b_k \geq 0$. For real eigenvalues, the inequality (33) holds true if either ($\lambda_H \geq 0$ and $\lambda_K \geq 0$) or ($\lambda_H \leq 0$ and $\lambda_K \leq 0$). This shows that the scheme is unconditionally stable if $a_h a_k + b_h b_k \geq 0$, for complex eigenvalues and if either ($\lambda_H \geq 0$ and $\lambda_K \geq 0$) or ($\lambda_H \leq 0$ and $\lambda_K \leq 0$), for real eigenvalues. When $\theta = 0$, the inequality (33) becomes

$$\left|1 - \frac{\delta t\lambda_K}{\lambda_H}\right| \leq 1,$$

i.e.,

$$\delta t \leq \frac{2\lambda_H}{\lambda_K} \ \ and \ \ \frac{\lambda_H}{\lambda_K} \geq 0.$$

Thus for $\theta = 0$, the scheme is conditionally stable. The stability of scheme for the other values of can be investigate in a similar manner. The stability of the scheme and conditioning of the component matrices H, K of the matrix P depend on the weight parameter and the minimum distance between any two collocation points h in the domain set $[a, b]$.

4.2. GKS's equation

By considering the obtained matrix we have

$$[H + \delta t\theta K]e^{n+1} = [H - \delta t(1-\theta)K]e^n, \tag{35}$$

where $H = [A_d + A_b]A^{-1}$ and $K = [E + C + H + \sigma G]A^{-1}$ so,

$$e^{n+1} = Pe^n,$$

where $P = [H + \delta t\theta K]^{-1}[H - \delta t(1-\theta)K]$. This method is stable if $\| P \|_2 \leq 1$ or $\rho(P) \leq 1$, which is spectral radius of the matrix P. The stability is assured if all the eigenvalues of the matrix $[H + \delta t\theta K]^{-1}[H - \delta t(1-\theta)K]$ satisfy the following condition

$$\left| \frac{\lambda_H - \delta t(1-\theta)\lambda_K}{\lambda_H + \delta t\theta\lambda_K} \right| \leq 1, \tag{36}$$

where λ_H and $\lambda_K$are eigenvalues of the matrices H and K respectively. When $\theta = 0.5$, the inequality (32) becomes

$$\left| \frac{\lambda_H - 0.5\delta t\lambda_K}{\lambda_H + 0.5\delta t\lambda_K} \right| \leq 1. \tag{37}$$

In the case of complex eigenvalues $\lambda_H = a_h + ib_h$and $\lambda_K = a_k + ib_k$, where a_h, a_k, b_h and b_k are any real numbers, the inequality (37) takes the following form,

$$\left| \frac{(a_h - 0.5\delta t a_k) + i(b_h - 0.5\delta t b_k)}{(a_h - 0.5\delta t a_k) + i(b_h - 0.5\delta t b_k)} \right| \leq 1. \tag{38}$$

The inequality (38) is satisfied if $a_h a_k + b_h b_k \geq 0$. For real eigenvalues, the inequality (37) holds true if either ($\lambda_H \geq 0$ and $\lambda_K \geq 0$) or ($\lambda_H \leq 0$ and $\lambda_K \leq 0$). This shows that the scheme is unconditionally stable if $a_h a_k + b_h b_k \geq 0$, for complex eigenvalues and if either ($\lambda_H \geq 0$ and $\lambda_K \geq 0$) or ($\lambda_H \leq 0$ and $\lambda_K \leq 0$), for real eigenvalues. When $\theta = 0$, the inequality (37) becomes

$$\left|1 - \frac{\delta t \lambda_K}{\lambda_H}\right| \leq 1,$$

i.e.,

$$\delta t \leq \frac{2\lambda_H}{\lambda_K} \ \ and \ \ \frac{\lambda_H}{\lambda_K} \geq 0.$$

Thus for $\theta = 0$, the scheme is conditionally stable. The stability of scheme for the other values of can be investigate in a similar manner. The stability of the scheme and conditioning of the component matrices H, K of the matrix P depend on the weight parameter and the minimum distance between any two collocation points h in the domain set $[a, b]$.

5. Errors

In this section, two error norms is defined that will be used for showing the accuracy of the method as follows

$$L_2 = \parallel u - \tilde{u} \parallel_2 = \sqrt{h \sum_{j=1}^{N} |u - \tilde{u_j}|^2},$$
$$L_\infty = \parallel u - \tilde{u} \parallel_\infty = max|u_j - \tilde{u_j}|, \ 1 \leq j \leq N,$$

where $u, \ \tilde{u}$ are exact and approximate solution respectively.

6. Numerical Solution

In this section L_2 and L_∞ are obtained and shown in Tables and approximate solution of KS and GKS's equation is shown in Figures.

Example 1. *Consider Kuramoto-Sivashinsky (KS) equation as*

$$u_t + uu_x + u_{xx} + u_{xxxx} = 0, \tag{39}$$

with the exact solution

$$u(x,t) = c + \frac{15}{19}\sqrt{\frac{11}{19}}(-9tanh(k(x - ct - x_0)) + 11tanh^3(k(x - ct - x_0))), \tag{40}$$

and the initial condition

$$u(x,0) = f(x), \tag{41}$$

and the boundary conditions

$$u(a,t) = g_a(t), \;\; u(b,t) = g_b(t), t \geq 0, \tag{42}$$

where $c = 0.1$, $k = \frac{1}{2}\sqrt{\frac{11}{19}}$, $x_0 = -10$, $n = 20$, $a = -30$, $b = 30$, $\delta t = 0.00001$.

In Table 1, two kinds of error is calculated for $n = 20$, $a = -30$, $b = 30$, $\delta t = 0.00001$, $T = 0.0001, \ldots, 0.0009$. *Figure 1, indicates the shock profile wave propagation of KS equation in different time level of* $T = 0, \ldots, 100$ *for* $n = 200$, $a = -30$, $b = 30$, $\delta t = 0.1$.

Time	L_∞	L_2
0.0001	2.69492×10^{-5}	8.62302×10^{-5}
0.0002	5.39004×10^{-5}	1.72464×10^{-4}
0.0003	8.08535×10^{-5}	2.58702×10^{-4}
0.0004	1.07809×10^{-4}	3.44943×10^{-4}
0.0005	1.34766×10^{-4}	4.31187×10^{-4}
0.0006	1.61725×10^{-4}	5.17436×10^{-4}
0.0007	1.88686×10^{-4}	6.03688×10^{-4}
0.0008	2.15649×10^{-4}	6.89944×10^{-4}
0.0009	2.42614×10^{-4}	7.76203×10^{-4}

Table1: Errors for $n = 20$, $a = -30$, $b = 30$, $\delta t = 0.00001$, $T = 0.0001, \ldots, 0.0009$.

Example 2. *Consider Kuramoto-Sivashinsky (KS) equation as*

$$u_t + uu_x - u_{xx} + u_{xxxx} = 0, \tag{43}$$

with the exact solution

$$u(x,t) = c + \frac{15}{19\sqrt{19}}(-3tanh(k(x - ct - x_0)) + tanh^3(k(x - ct - x_0))), \tag{44}$$

and the initial condition

$$u(x,0) = f(x), \tag{45}$$

and the boundary conditions

$$u(a,t) = g_a(t), \;\; u(b,t) = g_b(t), t \geq 0, \tag{46}$$

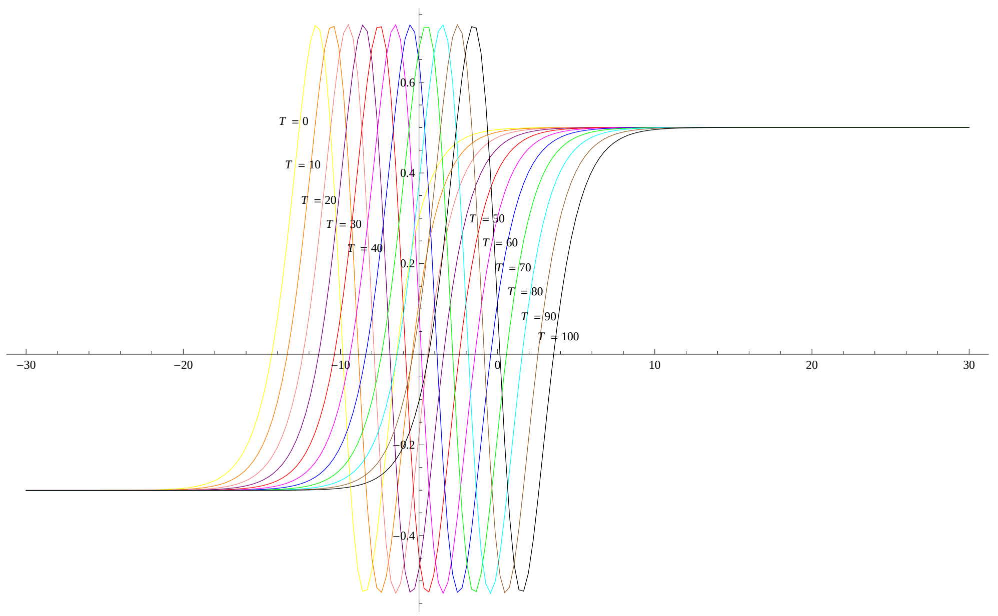

Figure 1: The shock profile wave propagation of KS equation for $\alpha = 1$.

where $c = 0.2$, $k = \frac{1}{2\sqrt{19}}$, $x_0 = -10$, $n = 20$, $a = -10$, $b = 10$, $\delta t = 0.0001$.

In Table 2, two kinds of error is calculated for $n = 20$, $a = -30$, $b = 30$, $\delta t = 0.0001$, $T = 0.0001, \ldots, 0.0009$. *Figure 2, indicates the shock profile wave propagation of KS equation in different time level of* $T = 0, \ldots, 80$ *for* $n = 100$, $a = -30$, $b = 30$, $\delta t = 0.1$.

Time	L_∞	L_2
0.0001	1.20375×10^{-5}	3.67788×10^{-5}
0.0002	2.40744×10^{-5}	7.35569×10^{-5}
0.0003	3.61109×10^{-5}	1.10334×10^{-4}
0.0004	4.81470×10^{-5}	1.47111×10^{-4}
0.0005	6.01825×10^{-5}	1.83886×10^{-4}
0.0006	7.22176×10^{-5}	2.20661×10^{-4}
0.0007	8.42523×10^{-5}	2.57435×10^{-4}
0.0008	9.62864×10^{-5}	2.94208×10^{-4}
0.0009	1.08320×10^{-4}	3.30981×10^{-4}

Table2: Errors for $n = 20$, $a = -30$, $b = 30$, $\delta t = 0.0001$, $T = 0.0001, \ldots, 0.0009$.

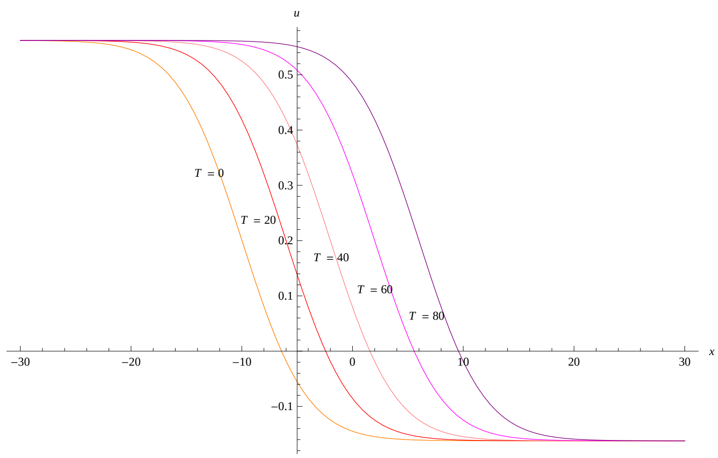

Figure 2: The shock profile wave propagation of KS equation for $\alpha = -1$.

Example 3. *Consider generalized Kuramoto-Sivashinsky (GKS) equation as*

$$u_t + uu_x + u_{xx} + \sigma u_{xxx} + u_{xxxx} = 0, \tag{47}$$

with the exact solution

$$\begin{aligned} u(x,t) &= c + 9 + -15(tanh(k(x-ct-x_0)) \\ &+ tanh^2(k(x-ct-x_0)) - tanh^3(k(x-ct-x_0))), \end{aligned} \tag{48}$$

and the initial condition

$$u(x,0) = f(x), \tag{49}$$

and the boundary conditions

$$u(a,t) = g_a(t), \;\; u(b,t) = g_b(t), t \geq 0, \tag{50}$$

where $c = 3$, $k = 0.5$, $\sigma = 4$, $x_0 = -10$, $n = 20$, $a = -10$, $b = 10$, $\delta t = 0.00001$.

In Table 3, two kinds of error is calculated for $n = 20$, $a = -10$, $b = 10$, $\delta t = 0.00001$, $T = 0.00001, \ldots, 0.00009$. *Figure 3, indicates solitonic solutions of GKS equation in different time level of* $T = 0, \ldots, 5$ *for* $n = 100$, $a = -20$, $b = 20$, $\delta t = 0.1$.

Time	L_∞	L_2
0.00001	*1.55374*$\times10^{-4}$	*2.93979*$\times10^{-4}$
0.00002	*3.10693*$\times10^{-4}$	*5.86548*$\times10^{-4}$
0.00003	*4.65958*$\times10^{-4}$	*8.79367*$\times10^{-4}$
0.00004	*6.21167*$\times10^{-4}$	*1.17221*$\times10^{-3}$
0.00005	*7.76322*$\times10^{-4}$	*1.46503*$\times10^{-3}$
0.00006	*9.31421*$\times10^{-4}$	*1.75782*$\times10^{-3}$
0.00007	*1.08647*$\times10^{-3}$	*2.05056*$\times10^{-3}$
0.00008	*1.24146*$\times10^{-3}$	*2.34326*$\times10^{-3}$
0.00009	*1.39639*$\times10^{-3}$	*2.63591*$\times10^{-3}$

Table3: Errors $n = 20$, $a = -10$, $b = 10$, $\delta t = 0.00001$, $T = 0.00001, \dots, 0.00009$.

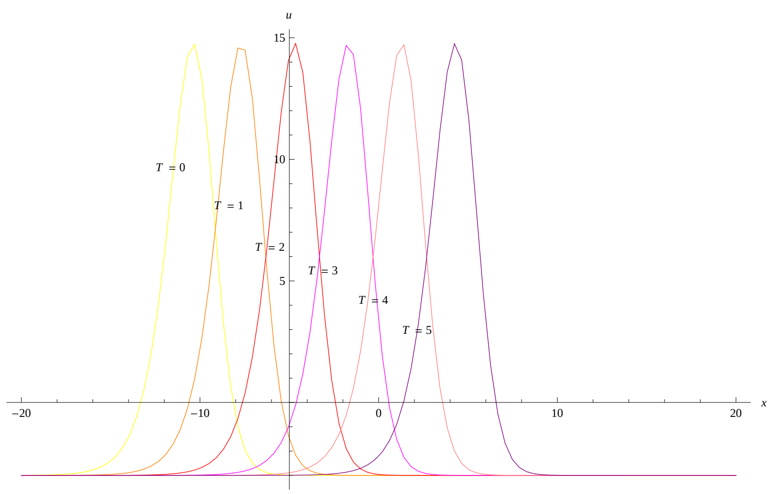

Figure 3: The solitonic solution of GKS equation.

Acknowledgements. The author thanks referees for reviewing this article.

References

[1] J. M. Hyman, B. Nicolaenko, *The Kuramoto-Sivashinky equation: a bridge between PDEs and dynamical systes*, Physica D. 18 (1986), 113- 126.

[2] Y. Zhang, L. Song, W. Axi, *Dynamical bifurcation for the Kuramoto-Sivashinsky equation*, Nonlinear. Anal-Theor. 74, 4 (2011), 1155-1163.

[3] S. Dubljevic, *Model predictive control of KuramotoSivashinsky equation with state and input constraints*, Chem. Eng. Sci. 65, 15 (2010), 4388-4396.

[4] Y. Bozhkov, S. *Dimas, Group classification and conservation laws for a two-dimensional generalized Kuramoto-Sivashinsky equation*, Nonlinear. Anal-Theor. 84 (2013), 117-135.

[5] L. Bo, Y. Jiang, *Large deviation for the nonlocal Kuramoto-Sivashinsky SPDE*, Nonlinear. Anal-Theor. 82 (2013), 100-114.

[6] B. Barker, M.A. Johnson, P. Noble, L.M Rodrigues, K. Zumbrun, *Stability of periodic Kuramoto-Sivashinsky waves*, Appl. Math. Lett. 25, 5 (2012) 824-829.

[7] D. Yang, *Kolmogorov equation associated to a stochastic KuramotoSivashinsky equation*, J. Funct. Anal. 263, 4 (2012), 869-895.

[8] F. Stenger, *Numerical methods based on sinc and analytical functions*, Springer, New York, 1993.

[9] S.U. Islam, S. Had, A. Ali, *A meshfree method for the numerical solution of RLW equation*, J. Comput. Appl. Math. 223 (2009), 997-1012.

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