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Hopf Structure for Poisson Enveloping Algebras

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Abstract. This work is to obtain a natural Hopf structure of the Poisson enveloping algebra $U(A)$ for a Poisson Hopf algebra A.

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Assume throughout the paper that k denotes a field of characteristic zero. Recall that $A = (A, \cdot, \{\cdot, \cdot\})$ is said to be a Poisson algebra if $(A, \cdot)$ is a commutative k-algebra and $(A, \{\cdot, \cdot\})$ is a Lie algebra such that

$$\{ab, c\} = a\{b, c\} + b\{a, c\}$$

for all $a, b, c \in A$. For every Poisson algebra A, there exists a unique Poisson enveloping algebra $U(A)$, which is a (associative) k-algebra, such that a k-vector space M is a Poisson A-module if and only if M is a $U(A)$-module (see [4, **1**, **5** and **6**]). The main purpose of this paper is to see that if A is also a Hopf algebra with Hopf structure compatible with the given Poisson structure (in this case, A is called a Poisson Hopf algebra) then $U(A)$ is a Hopf algebra.

Throughout the paper that, for an algebra B, B_L will be the Lie algebra B with Lie bracket $[a, b] = ab - ba$ for all $a, b \in B$.

Let us review a definition of Poisson enveloping algebra (see [4, **3**]):

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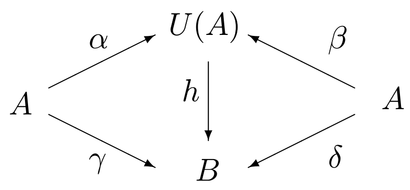
Definition 1. *For a Poisson algebra A, a triple $(U(A), \alpha, \beta)$, where $U(A)$ is an algebra, $\alpha : A \longrightarrow U(A)$ is an algebra homomorphism and $\beta : A \longrightarrow U(A)_L$ is a Lie homomorphism such that*

$$\alpha(\{a, b\}) = [\beta(a), \alpha(b)], \quad \beta(ab) = \alpha(a)\beta(b) + \alpha(b)\beta(a)$$

for all $a, b \in A$, is called the Poisson enveloping algebra for A if $(U(A), \alpha, \beta)$ satisfies the following; if B is a k-algebra, γ is an algebra homomorphism from A into B and δ is a Lie homomorphism from $(A, \{\cdot, \cdot\})$ into B_L such that

$$\gamma(\{a, b\}) = [\delta(a), \gamma(b)], \quad \delta(ab) = \gamma(a)\delta(b) + \gamma(b)\delta(a)$$

for all $a, b \in A$, then there exists a unique algebra homomorphism h from $U(A)$ into B such that $h\alpha = \gamma$ and $h\beta = \delta$.



For every Poisson algebra A over k, note that there exists a unique Poisson enveloping algebra $(U(A), \alpha, \beta)$ up to isomorphic, that $U(A)$ is generated by $\alpha(A)$ and $\beta(A)$ by [4, *proof of* **5**] *and that $\beta(1) = 0$.*

Definition 2. (see [3, 3.1.3]) *A Poisson algebra A is said to be a Poisson Hopf algebra if A is also a Hopf algebra $(A, \iota, \mu, \epsilon, \Delta, S)$ over k such that both structures are compatible in the sense that*

$$\Delta(\{a, b\}) = \{\Delta(a), \Delta(b)\}_{A\otimes A}$$

for all $a, b \in A$, where the Poisson bracket $\{\cdot, \cdot\}_{A\otimes A}$ on $A \otimes A$ is defined by

$$\{a \otimes a', b \otimes b'\}_{A\otimes A} = \{a, b\} \otimes a'b' + ab \otimes \{a', b'\}$$

for all $a, a', b, b' \in A$.

For example, every coordinate ring of Poisson Lie group is a Poisson Hopf algebra.

For Poisson algebras A and B, an algebra homomorphism $\phi : A \longrightarrow B$ is said to be a Poisson homomorphism (respectively, anti-homomorphism) if ϕ satisfies the rule

$$\phi(\{a, b\}) = \{\phi(a), \phi(b)\} \quad \text{(respectively, } \phi(\{a, b\}) = \{\phi(b), \phi(a)\})$$

for all $a, b \in A$.

Lemma 3. *If $(A, \iota, \mu, \epsilon, \Delta, S)$ is a Poisson Hopf algebra then the counit ϵ is a Poisson homomorphism and the antipode S is a Poisson anti-automorphism.*

Proof. [3, Remark 3.1.4] □

Lemma 4. *If γ and δ are k-linear maps from a Poisson algebra A into a k-algebra B such that*

$$\gamma(\{a,b\}) = [\delta(a), \gamma(b)], \quad \delta(ab) = \gamma(a)\delta(b) + \gamma(b)\delta(a)$$

for all $a, b \in A$, then

$$\gamma(\{a,b\}) = [\gamma(a), \delta(b)], \quad \delta(ab) = \delta(a)\gamma(b) + \delta(b)\gamma(a).$$

Proof. Since

$$\begin{aligned}\gamma(\{a,b\}) + \delta(ab) &= \delta(a)\gamma(b) + \gamma(a)\delta(b)\\ \gamma(\{b,a\}) + \delta(ba) &= \delta(b)\gamma(a) + \gamma(b)\delta(a)\end{aligned}$$

we have

$$\begin{aligned}2\delta(ab) &= \delta(a)\gamma(b) + \delta(b)\gamma(a) + \gamma(a)\delta(b) + \gamma(b)\delta(a)\\ &= \delta(a)\gamma(b) + \delta(b)\gamma(a) + \delta(ab)\\ 2\gamma(\{a,b\}) &= \delta(a)\gamma(b) - \delta(b)\gamma(a) + \gamma(a)\delta(b) - \gamma(b)\delta(a)\\ &= \gamma(a)\delta(b) - \delta(b)\gamma(a) + \gamma(\{a,b\})\end{aligned}$$

by adding and subtracting the above two formulas. Hence we have the conclusion. □

Lemma 5. *Let $(U(A), \alpha, \beta)$ be the Poisson enveloping algebra for a Poisson algebra A. Then*
(i) *$\alpha \otimes \alpha : A \otimes A \longrightarrow U(A) \otimes U(A)$ is an algebra homomorphism.*
(ii) *$\alpha \otimes \beta + \beta \otimes \alpha : A \otimes A \longrightarrow (U(A) \otimes U(A))_L$ is a Lie homomorphism.*

Proof. (i) It is clear since α is an algebra homomorphism.
(ii) By Lemma 4, for $a, a', b, b' \in A$,

$$\begin{aligned}&(\alpha\otimes\beta+\beta\otimes\alpha)(\{a\otimes a', b\otimes b'\})\\ &\qquad - [(\alpha\otimes\beta+\beta\otimes\alpha)(a\otimes a'), (\alpha\otimes\beta+\beta\otimes\alpha)(b\otimes b')]\\ &= (\alpha\otimes\beta+\beta\otimes\alpha)(ab\otimes\{a',b'\} + \{a,b\}\otimes a'b')\\ &\qquad - [(\alpha\otimes\beta+\beta\otimes\alpha)(a\otimes a'), (\alpha\otimes\beta+\beta\otimes\alpha)(b\otimes b')]\\ &= \alpha(ab)\otimes\beta(\{a',b'\}) + \alpha(\{a,b\})\otimes\beta(a'b') + \beta(ab)\otimes\alpha(\{a',b'\})\\ &\qquad + \beta(\{a,b\})\otimes\alpha(a'b') - [\alpha(a)\otimes\beta(a'), \alpha(b)\otimes\beta(b')]\\ &\qquad - [\alpha(a)\otimes\beta(a'), \beta(b)\otimes\alpha(b')] - [\beta(a)\otimes\alpha(a'), \alpha(b)\otimes\beta(b')]\\ &\qquad - [\beta(a)\otimes\alpha(a'), \beta(b)\otimes\alpha(b')]\\ &= \alpha(ab)\otimes[\beta(a'),\beta(b')] + [\beta(a),\alpha(b)]\otimes(\alpha(a')\beta(b') + \alpha(b')\beta(a'))\\ &\qquad + (\alpha(a)\beta(b) + \alpha(b)\beta(a))\otimes[\beta(a'),\alpha(b')] + [\beta(a),\beta(b)]\otimes\alpha(a'b')\\ &\qquad - \alpha(a)\alpha(b)\otimes[\beta(a'),\beta(b')] - \alpha(a)\beta(b)\otimes\beta(a')\alpha(b')\\ &\qquad + \beta(b)\alpha(a)\otimes\alpha(b')\beta(a') - \beta(a)\alpha(b)\otimes\alpha(a')\beta(b')\\ &\qquad + \alpha(b)\beta(a)\otimes\beta(b')\alpha(a') - [\beta(a),\beta(b)]\otimes\alpha(a')\alpha(b')\\ &= -\alpha(b)\beta(a)\otimes[\alpha(a'),\beta(b')] + [\beta(a),\alpha(b)]\otimes\alpha(b')\beta(a')\\ &\qquad - [\alpha(a),\beta(b)]\otimes\alpha(b')\beta(a') + \alpha(b)\beta(a)\otimes[\beta(a'),\alpha(b')]\\ &= -\alpha(b)\beta(a)\otimes\alpha(\{a',b'\}) + \alpha(\{a,b\})\otimes\alpha(b')\beta(a')\\ &\qquad - \alpha(\{a,b\})\otimes\alpha(b')\beta(a') + \alpha(b)\beta(a)\otimes\alpha(\{a',b'\})\\ &= 0.\end{aligned}$$

Hence $\alpha \otimes \beta + \beta \otimes \alpha$ is a Lie homomorphism. □

Lemma 6. *Let A and B be Poisson algebras and let C be an algebra. If $\phi : A \longrightarrow B$ is a Poisson homomorphism, $\alpha : B \longrightarrow C$ is an algebra homomorphism and $\beta : B \longrightarrow C_L$ is a Lie homomorphism such that*

$$\alpha(\{b_1, b_2\}) = [\beta(b_1), \alpha(b_2)], \quad \beta(b_1 b_2) = \alpha(b_1)\beta(b_2) + \alpha(b_2)\beta(b_1)$$

for all $b_1, b_2 \in B$ then $\alpha\phi : A \longrightarrow C$ is an algebra homomorphism and $\beta\phi : A \longrightarrow C_L$ is a Lie homomorphism such that

$$\begin{aligned} \alpha\phi(\{a_1, a_2\}) &= [\beta\phi(a_1), \alpha\phi(a_2)] \\ \beta\phi(a_1 a_2) &= \alpha\phi(a_1)\beta\phi(a_2) + \alpha\phi(a_2)\beta\phi(a_1) \end{aligned}$$

for all $a_1, a_2 \in A$.

Proof. Straightforward. □

Lemma 7. *Let $(U(A), \alpha, \beta)$ be the Poisson enveloping algebra for a Poisson algebra A. Then $(U(A) \otimes U(A), \alpha \otimes \alpha, \alpha \otimes \beta + \beta \otimes \alpha)$ is the Poisson enveloping algebra for $A \otimes A$.*

Proof. It is straightforward to see that

$$\begin{aligned} (\alpha \otimes \alpha)(\{a \otimes a', b \otimes b'\}) &= [(\alpha \otimes \beta + \beta \otimes \alpha)(a \otimes a'), (\alpha \otimes \alpha)(b \otimes b')] \\ (\alpha \otimes \beta + \beta \otimes \alpha)((a \otimes a')(b \otimes b')) &= (\alpha \otimes \alpha)(a \otimes a')(\alpha \otimes \beta + \beta \otimes \alpha)(b \otimes b') \\ &\quad + (\alpha \otimes \alpha)(b \otimes b')(\alpha \otimes \beta + \beta \otimes \alpha)(a \otimes a'). \end{aligned}$$

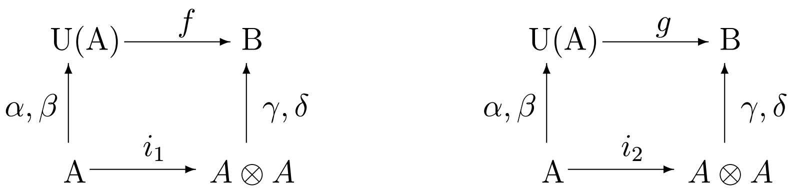
Let i_1 and i_2 be the Poisson homomorphisms from A into $A \otimes A$ defined by

$$\begin{aligned} i_1 : A \longrightarrow A \otimes A, &\qquad i_1(a) = a \otimes 1 \\ i_2 : A \longrightarrow A \otimes A, &\qquad i_2(a) = 1 \otimes a \end{aligned}$$

for all $a \in A$. Given an algebra B, let μ_B be the multiplication map on B. If γ is an algebra homomorphism from $A \otimes A$ into B and δ is a Lie homomorphism from $A \otimes A$ into B_L such that

$$\begin{aligned} \gamma(\{a \otimes a', b \otimes b'\}) &= [\delta(a \otimes a'), \gamma(b \otimes b')] \\ \delta((a \otimes a')(b \otimes b')) &= \gamma(a \otimes a')\delta(b \otimes b') + \gamma(b \otimes b')\delta(a \otimes a') \end{aligned}$$

for all $a, a', b, b' \in A$, then there exist algebra homomorphisms f, g from $U(A)$ into B such that $f\alpha = \gamma i_1, f\beta = \delta i_1, g\alpha = \gamma i_2, g\beta = \delta i_2$ by Lemma 6.



Moreover we have $\delta i_1(a)\gamma i_2(a') = \gamma i_2(a')\delta i_1(a)$ for all $a, a' \in A$ since

$$\begin{aligned}[\delta i_1(a), \gamma i_2(a')] &= \gamma(\{a \otimes 1, 1 \otimes a'\}) \\ &= \gamma(a \otimes \{1, a'\} + \{a, 1\} \otimes a') \\ &= 0.\end{aligned}$$

Hence we have

$$\begin{aligned}\mu_B(f \otimes g)(\alpha \otimes \alpha)(a \otimes a') &= f\alpha(a)g\alpha(a') = \gamma i_1(a)\gamma i_2(a') \\ &= \gamma(i_1(a)i_2(a')) = \gamma(a \otimes a') \\ \mu_B(f \otimes g)(\alpha \otimes \beta + \beta \otimes \alpha)(a \otimes a') &= f\alpha(a)g\beta(a') + f\beta(a)g\alpha(a') \\ &= \gamma i_1(a)\delta i_2(a') + \delta i_1(a)\gamma i_2(a') \\ &= \gamma i_1(a)\delta i_2(a') + \gamma i_2(a')\delta i_1(a) \\ &= \delta(i_1(a)i_2(a')) \\ &= \delta(a \otimes a')\end{aligned}$$

for all $a, a' \in A$. Thus $\mu_B(f \otimes g)$ is an algebra homomorphism such that

$$\mu_B(f \otimes g)(\alpha \otimes \alpha) = \gamma, \quad \mu_B(f \otimes g)(\alpha \otimes \beta + \beta \otimes \alpha) = \delta.$$

If $h : U(A \otimes A) \longrightarrow B$ is an algebra homomorphism such that

$$h(\alpha \otimes \alpha) = \gamma, \quad h(\alpha \otimes \beta + \beta \otimes \alpha) = \delta$$

then

$$\begin{aligned}\mu_B(f \otimes g)(\alpha(a) \otimes 1) &= h(\alpha \otimes \alpha)(a \otimes 1) = h(\alpha(a) \otimes 1) \\ \mu_B(f \otimes g)(1 \otimes \alpha(a)) &= h(\alpha \otimes \alpha)(1 \otimes a) = h(1 \otimes \alpha(a)) \\ \mu_B(f \otimes g)(1 \otimes \beta(a)) &= h(\alpha \otimes \beta + \beta \otimes \alpha)(1 \otimes a) = h(1 \otimes \beta(a)) \\ \mu_B(f \otimes g)(\beta(a) \otimes 1) &= h(\alpha \otimes \beta + \beta \otimes \alpha)(a \otimes 1) = h(\beta(a) \otimes 1)\end{aligned}$$

for all $a \in A$, hence we have $\mu_B(f \otimes g) = h$ since $U(A)$ is generated by $\alpha(A)$ and $\beta(A)$. It completes the proof by Lemma 5. □

Lemma 8. *Let $(U(A), \alpha_A, \beta_A)$ and $(U(B), \alpha_B, \beta_B)$ be Poisson enveloping algebras for Poisson algebras A and B respectively. If $\phi : A \longrightarrow B$ is a Poisson homomorphism then there exists a unique algebra homomorphism $U(\phi) : U(A) \longrightarrow U(B)$ such that $U(\phi)\alpha_A = \alpha_B\phi$ and $U(\phi)\beta_A = \beta_B\phi$.*

$$\begin{array}{ccc} U(A) & \xrightarrow{U(\phi)} & U(B) \\ {\scriptstyle \alpha_A, \beta_A}\uparrow & & \uparrow{\scriptstyle \alpha_B, \beta_B} \\ A & \xrightarrow{\phi} & B \end{array}$$

Proof. It follows immediately from the definition for Poisson enveloping algebra and Lemma 6. □

Let $A = (A, \cdot, \{\cdot, \cdot\})$ be a Poisson algebra. Define a k-bilinear map $\{\cdot, \cdot\}_1$ on A by

$$\{a, b\}_1 = \{b, a\}$$

for all $a, b \in A$. Then $A_1 = (A, \cdot, \{\cdot, \cdot\}_1)$ is a Poisson algebra. For an algebra B, we denote by $B^{op} = (B, \circ)$ the opposite algebra of B.

Proposition 9. *Let $(U(A), \alpha, \beta)$ be the Poisson enveloping algebra for a Poisson algebra A. Then $(U(A)^{op}, \alpha, \beta)$ is the Poisson enveloping algebra for A_1.*

Proof. Clearly, α is an algebra homomorphism from A_1 into $U(A)^{op}$ since A_1 is commutative and β is a Lie homomorphism from A_1 into $U(A)^{op}_L$. Moreover, by Lemma 4, we have

$$\begin{aligned}\alpha(\{a, b\}_1) = \alpha(\{b, a\}) = [\alpha(b), \beta(a)] = \beta(a) \circ \alpha(b) - \alpha(b) \circ \beta(a)\\ \beta(ab) = \beta(a)\alpha(b) + \beta(b)\alpha(a) = \alpha(a) \circ \beta(b) + \alpha(b) \circ \beta(a)\end{aligned}$$

for all $a, b \in A_1$. If B is an algebra, $\gamma : A_1 \longrightarrow B$ is an algebra homomorphism and $\delta : A_1 \longrightarrow B_L$ is a Lie homomorphism such that

$$\gamma(\{a, b\}_1) = [\delta(a), \gamma(b)] \quad \text{and} \quad \delta(ab) = \gamma(a)\delta(b) + \gamma(b)\delta(a)$$

for all $a, b \in A_1$, then $\gamma : A \longrightarrow B^{op}$ is an algebra homomorphism and $\delta : A \longrightarrow B^{op}_L$ is a Lie homomorphism such that

$$\begin{aligned}\gamma(\{a, b\}) = \gamma(\{b, a\}_1) = [\gamma(b), \delta(a)] = \delta(a) \circ \gamma(b) - \gamma(b) \circ \delta(a)\\ \delta(ab) = \delta(a)\gamma(b) + \delta(b)\gamma(a) = \gamma(a) \circ \delta(b) + \gamma(b) \circ \delta(a)\end{aligned}$$

for all $a, b \in A$ by Lemma 4. Hence there is a unique algebra homomorphism h from $U(A)$ into B^{op} such that $h\alpha = \gamma$ and $h\beta = \delta$ and so $h : U(A)^{op} \longrightarrow B$ is a unique algebra homomorphism such that $h\alpha = \gamma$ and $h\beta = \delta$. Thus $(U(A)^{op}, \alpha, \beta)$ is the Poisson enveloping algebra for A_1. □

Theorem 10. *If $(A, \iota, \mu, \epsilon, \Delta, S)$ is a Poisson Hopf algebra then*

$$(U(A), \iota_{U(A)}, \mu_{U(A)}, U(\epsilon), U(\Delta), U(S))$$

is a Hopf algebra such that

$$\begin{aligned}U(\Delta)\alpha &= (\alpha \otimes \alpha)\Delta & \qquad U(\Delta)\beta &= (\alpha \otimes \beta + \beta \otimes \alpha)\Delta\\ U(\epsilon)\alpha &= \epsilon & U(\epsilon)\beta &= 0\\ U(S)\alpha &= \alpha S & U(S)\beta &= \beta S.\end{aligned}$$

Proof. Since Δ is a Poisson homomorphism and $(U(A)\otimes U(A), \alpha\otimes\alpha, \alpha\otimes\beta+\beta\otimes\alpha)$ is the Poisson enveloping algebra of $A\otimes A$ by Lemma 7, there exists an algebra homomorphism $U(\Delta): U(A)\longrightarrow U(A)\otimes U(A)$ such that

$$U(\Delta)\alpha = (\alpha\otimes\alpha)\Delta,\quad U(\Delta)\beta = (\alpha\otimes\beta+\beta\otimes\alpha)\Delta$$

by Lemma 8. Similarly, there exists an algebra homomorphism $U(\epsilon)$ from $U(A)$ into k such that $U(\epsilon)\alpha=\epsilon$, $U(\epsilon)\beta=0$ since $(k, \mathrm{id}_k, 0)$ is the Poisson enveloping algebra of the field k with trivial Poisson bracket. Since the antipode S is a Poisson homomorphism from A into A_1 by Lemma 3, there is an algebra homomorphism $U(S): U(A)\longrightarrow U(A)^{op}$ such that $U(S)\alpha=\alpha S$ and $U(S)\beta=\beta S$ by Lemma 8 and Proposition 9. It is verified routinely that $(U(A), \iota_{U(A)}, \mu_{U(A)}, U(\epsilon), U(\Delta), U(S))$ is a Hopf algebra. □

Example 11. Let L be a finite dimensional Lie algebra over k with Lie bracket $[\cdot,\cdot]$ and let $\mathcal{S}(L)$ be the symmetric algebra of L. Fix a k-basis $x_1,\dots,x_n$ of L. Note that $\mathcal{S}(L)$ is the commutative polynomial ring $k[x_1,\dots,x_n]$. Then, by [1, 2.8.7] or [2, Example 1], $\mathcal{S}(L)$ is a Poisson Hopf algebra with structure

$$\{a,b\}=[a,b],\ \Delta(a)=a\otimes 1+1\otimes a,\ \epsilon(a)=0,\ S(a)=-a$$

for all $a,b\in L$. In fact, it is verified easily using the induction on degree of homogeneous elements of $\mathcal{S}(L)$ that

$$\begin{aligned}\Delta(\{x,y\}) &= \{\Delta(x),\Delta(y)\}\\ \epsilon(\{x,y\}) &= 0\\ S(\{x,y\}) &= \{S(y),S(x)\}\end{aligned}$$

for all $x,y\in\mathcal{S}(L)$. Observe that the Poisson enveloping algebra $U(\mathcal{S}(L))=(U(\mathcal{S}(L)),\alpha,\beta)$ is the algebra generated by $x_1,\dots,x_n,y_1,\dots,y_n$ subject to the relation

$$x_ix_j=x_jx_i,\quad y_iy_j=y_jy_i+\psi([x_i,x_j]),\quad x_iy_j=y_jx_i+[x_i,x_j]$$

for all $i,j=1,\dots,n$ and, α and β are given by $\alpha(x_i)=x_i$, $\beta(x_i)=y_i$, respectively, where $\psi: L\longrightarrow U(\mathcal{S}(L))$ is a k-linear map defined by $\psi(x_i)=y_i$ for all $i=1,\dots,n$. By Theorem 10, the Poisson enveloping algebra $U(\mathcal{S}(L))$ is a Hopf algebra with Hopf structure

$$\begin{aligned}\Delta(x_i) &= x_i\otimes 1+1\otimes x_i & \Delta(y_i) &= y_i\otimes 1+1\otimes y_i\\ \epsilon(x_i) &= 0 & \epsilon(y_i) &= 0\\ S(x_i) &= -x_i & S(y_i) &= -y_i\end{aligned}$$

for all $i=1,\dots,n$.

The Poisson enveloping algebra $U(\mathcal{S}(L))$ contains the universal enveloping algebra of L as a subalgebra. Let U be the subalgebra of $U(\mathcal{S}(L))$ generated by $y_1,\dots,y_n$ and let $j: L\longrightarrow U$ be a k-linear map defined by $j(x_i)=y_i$ for all $i=1,\dots,n$. Then (U,j) is the universal enveloping algebra of L.

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