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REGULARIZATION FOR A CLASS OF BACKWARD PARABOLIC PROBLEMS

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Abstract. The backward Cauchy problem $u_t + Au(t) = 0, u(T) = f$, where A is a positive self-adjoint unbounded operator,which has continuous spectrum and f is a given function being given is regularized by the well-posed problem the truncation method. The new error estimates of the regularized solution are obtained. The main purpose of this paper is to improve the earlier results by [2, 19].

1. Introduction

Let H be a Hilbert space. For a positive number T, we shall consider the problem of finding the function $u : [0,T] \to H$ from the system

$$\begin{cases} u_t + Au = 0, \ 0 < t < T \\ u(T) = f \end{cases} \tag{1.1}$$

for some prescribed final value f in H. The operator A is a positive self-adjoint operator such that $0 \in \rho(A)$. This problem is well known to be severely ill-posed and regularization methods for it are required.

The case A be a self-adjoint operator having the discrete spectrum on H has been considered by many authors, using different approaches. Such authors as Latt'es and Lions [9],Miller [10], Payne [12] have approximated (1) by perturbing the operator A. This method is called Quasi-reversibility method (QR). The main ideas of the method is of adding a "corrector" into the main equation. In fact, they considered the problem

$$\begin{cases} u_t + Au - \epsilon A^* Au = 0, \ 0 < t < T \\ u(T) = f \end{cases} \tag{1.2}$$

The stability magnitude of the method are of order $e^{c\epsilon^{-1}}$. In [7], the problem is approximated with

$$\begin{cases} u_t + Au + \epsilon Au_t = 0, \ 0 < t < T \\ u(T) = f \end{cases} \tag{1.3}$$

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Ames and Hughes [1] gave a survey about an association between the operator-theoretic methods and the QR method to treat the abstract Cauchy problem

$$\frac{du}{dt} = Au, \ u(T) = \chi, \quad 0 < t < T.$$

The authors considered the problem in both the Hilbert space and in the Banach space. They also gave many structural stability results. Very recently, using the QR method, Yongzhong Huang and Quan Zheng, in [17], considered the problem in an abstract setting, i.e., $-A$ is the generator of an analytic semigroup in a Banach space.

In [14], Showalter presented a different method called the quasiboundary value (QBV) method to regularize that linear homogeneous problem which gave a stability estimate better than the one of discussed methods. The main ideas of the method is of adding an appropriate "corrector" into the final data. Using the method, Clark-Oppenheimer, in [3]in regularizing the problem (1.1) by the nonlocal boundary value problem

$$\begin{cases} u_t + Au = 0, \ 0 < t < T \\ \epsilon u(0) + u(T) = f \end{cases} \tag{1.4}$$

Very recently, Denche-Bessila in [18], regularized the backward problem by pertubing the final condition as follows

$$\begin{cases} u_t + Au = 0, \ 0 < t < T \\ \epsilon u_t(0) + u(T) = f \end{cases} \tag{1.5}$$

In our knowledge, the case A has discrete spectrum has been treated in many recent papers, such as [16, 18]. However, the literature on the homogeneous case of the problem in the case A has continuous spectrum are quite scarce. For some related works on this type of such problem, we refer the reader to N.Boussetila and F. Rebbani [2], Denche and S. Djezzar [19].

In the present paper, we shall use new truncated method to extend the continuous dependence results of [2, 19]. Recently,the truncated regularization method has been effectively applied to solve the sideways heat equation [4], a more general sideways parabolic equation [5] and backward heat [6]. This regularization method is rather simple and convenient for dealing with some ill-posed problems. However, as far as we know, there are not any results of truncated method for treating the problem (1) until now. Moreover, we establish some new error estimates including the order of Holder type. Especially, the convergence of the approximate solution at $t = 0$ is also proved.

This paper is organized as follows. In the next section, for ease of the reading, we summarize some well-known facts in semigroup of operator. The stability estimates of the regularized solution will be presented in Section 3.

Before going to the details of next sections, we shall give the precise formula of the operator $S(t)$. We assume that H is a separable Hilbert space and A is self-adjoint and that 0 is in the resolvent set of A. $S(t)$ is the compact contraction semi group generated by $-A$. We denote by $\{E_\lambda, \ \lambda \geq 0\}$ the spectral resolution of the identity associated to A. Then $S(t) = e^{-tA} = \int_0^\infty e^{-t\lambda}\, dE_\lambda \in \mathcal{L}(H)$, $t \geq 0$, the

C_0-semigroup generated by $-A$. Then from [13], we get

$$Au = \int_0^{+\infty} \lambda dE_\lambda u \tag{1.6}$$

for all $u \in D(A)$. In this connection, $u \in D(A)$ iff the integral (1.6) exists,i.e.,

$$\int_0^{+\infty} \lambda^2 d\|E_\lambda u\|^2 < \infty.$$

2. The main results

A solution of (1.1) on the interval $[0,T]$ is a function $u \in \mathcal{C}([0,T];H)\cap\mathcal{C}^1((0,T);H)$ such that for all $t \in (0,T)$, $u(t) \in \mathcal{D}(A)$ and $u(T) = f$ holds. It is useful to know exactly the admissible set for which (1.1) has a solution. The following lemma gives an answer to this question.

Lemma 2.1. *Problem (1.1) has a solution if and only if*

$$\int_0^{\infty} e^{2\lambda T} d\|E_\lambda f\|^2 < \infty$$

and its unique solution is represented by

$$u(t) = e^{(T-t)A} f. \tag{2.1}$$

If the problem (1.1) admits a solution u then this solution can be represented by

$$u(t) \;=\; e^{(T-t)A} f = \int_0^{\infty} e^{\lambda(T-t)} dE_\lambda f. \tag{2.2}$$

Since $t < T$, we know from (2.2) that the terms $e^{-(t-T)\lambda}$ is the unstability cause. So, to regularize problem (2.2),we should replace it by the better terms. In [19], the authors replaced $e^{-(t-T)\lambda}$ by the better term $\frac{e^{-t\lambda}}{\epsilon\lambda + e^{-T\lambda}}$. In this paper, we hope to recover the stability of problem (2.2) by filtering the high frequencies with suitable method. The essence of our regularization method is just to eliminate all high frequencies from the solution, and instead consider (2.2) only for $\lambda \le \beta$, where β is an appropriate positive constant depend on ϵ which will be selected appropriately as regularization parameter. Let f and f_ϵ denote the exact and measured data at t = T, respectively, which satisfy

$$\|f - f_\epsilon\| \le \epsilon.$$

Hence, the ill-posed problem (1.1) can be approximated by the problem

$$u_\epsilon(t) \;=\; \int_0^{\infty} e^{\lambda(T-t)} \chi_{[0,\beta]} dE_\lambda f \tag{2.3}$$

where $\chi_{[a,b]}$ is the characteristic function of interval $[a,b]$ for $a < b$.
The approximated solution v_ϵ corresponding to the final value f_ϵ is given the form

$$v_\epsilon(t) \;=\; \int_0^{\infty} e^{\lambda(T-t)} \chi_{[0,\beta]} dE_\lambda f_\epsilon. \tag{2.4}$$

For clarity, from now on, we denote the solution of (1) by $u(t)$, and the solution of (2.4) by $v_\epsilon(t)$. Our first main theorem is the following

Theorem 2.2. *The solution defined in* (2.4) *depends continuously (in* $C([0,T];H)$*) on* f_ϵ*. Let* v_ϵ *and* w_ϵ *be two solutions of problem* (2.4) *corresponding to the final values* f_ϵ *and* g_ϵ *respectively, then*

$$\|v_\epsilon(t) - w_\epsilon(t)\| \le e^{(T-t)\beta}\|f_\epsilon - g_\epsilon\|.$$

Remark. *1) If* $\beta = \frac{1}{T}\ln(\frac{T}{\epsilon(1+\ln(\frac{T}{\epsilon}))})$*, the stability magnitude is*

$$E_1(\epsilon,t) = C_1 e^{(T-t)\beta} = C_1 \epsilon^{\frac{t}{T}-1}\left(T^{-1}\ln(Te\epsilon^{-1})\right)^{\frac{t}{T}-1}.$$

Note that the stability order in [19] *is the form* $E_2(\epsilon) = C_2\frac{T}{\epsilon(1+\ln(\frac{T}{\epsilon}))}$*. Comparing* $E_1(\epsilon,t)$ *and* $E_2(\epsilon,t)$*, we see that the stability order of our method is less than its in* [19].

2. Using Theorem 2.2, we have the error

$$\|u_\epsilon(t) - v_\epsilon(t)\| \le e^{(T-t)\beta}\|f - f_\epsilon\| \le e^{(T-t)\beta}\epsilon. \tag{2.5}$$

Theorem 2.3. *Let* $u \in C([0,T];H)$ *be a solution of* (1.1)*. Then*

$$\|u(t) - v_\epsilon(t)\| \le e^{(T-t)\beta}\epsilon + e^{-t\beta}\|u(0)\|, \ \forall t \in (0,T]. \tag{2.6}$$

Remark. *1. If we choose* $\beta = \frac{1}{T}\ln(\frac{1}{\epsilon})$ *then the estimates* (2.6) *becomes*

$$\|u(t) - u_\epsilon(t)\| \le \epsilon^{\frac{t}{T}}(1 + \|u(0)\|). \tag{2.7}$$

This error is also given by Clark and Oppenheimer [3]*, Tautenhahn* [20].

2. The error in $t = 0$ *is not considered in* (2.6)*. In the next Theorem, we shall establish some estimates which convergences to zero in* $t = 0$.

Theorem 2.4. *Assume that* u *has the eigenfunction expansion* $u(t) = \int_0^\infty dE_\lambda u(t)$.

a) Assume that there exist some positive constants p *and* I_1 *such that*

$$\int_0^\infty \lambda^{2p} d\|E_\lambda u(t)\|^2 < I_1^2. \tag{2.8}$$

If we choose $\beta = \frac{a}{T}\ln(\frac{1}{\epsilon}), (0 < a < 1)$*, then for every* $t \in [0,T]$

$$\|u(t) - v_\epsilon(t)\| \le \epsilon^{\frac{at}{T}+1-a} + (\frac{T}{a})^p I_1\left(\ln(\frac{1}{\epsilon})\right)^{-p}. \tag{2.9}$$

b) Assume that there exist some positive constants q *and* I_2 *such that*

$$\int_0^\infty e^{2q\lambda} d\|E_\lambda u(t)\|^2 < I_2^2. \tag{2.10}$$

If we choose $\beta = \frac{1}{T+q}\ln(\frac{1}{\epsilon})$*, then for every* $t \in [0,T]$

$$\|u(t) - v_\epsilon(t)\| \le \epsilon^{\frac{q}{T+q}}\left(\epsilon^{\frac{t}{T+q}} + I_2\right). \tag{2.11}$$

Remark. *1. We know that the exact solution* u *of (1.1) is unknown. Therefore, in practice, the assumptions a) and b) in Theorem 2.4 are very difficult to check. To improve this, we give the different conditions on the known function* f *as follows*

$$\int_0^\infty \lambda^{2p} e^{2(T-t)\lambda} d\|E_\lambda f\|^2 < I_3^2. \tag{2.12}$$

and

$$\int_0^\infty e^{2(T-t+q)\lambda} d\|E_\lambda f\|^2 < I_4^2. \tag{2.13}$$

where I_3 and I_4 are the positive numbers. Then, by similar way, we obtain the same convergence result.

2. One superficial advantage of this method is that there is a error estimation in the original time $t = 0$, which is not given in [22]. *We have the following estimate in $t = 0$*

$$\|u(0) - v_\epsilon(0)\| \le \epsilon^{1-a} + (\frac{T}{a})^p I_1 \left(\ln(\frac{1}{\epsilon})\right)^{-p}. \tag{2.14}$$

and

$$\|u(0) - v_\epsilon(0)\| \le \epsilon^{\frac{q}{T+q}}(1 + I_2). \tag{2.15}$$

3. It follows from (2.14) *that if $\epsilon \to 0$ then the second term on the righthand side of the inequality approaches zero with a logarithmic speed, and the first one as a power.So, the terms in* (2.14) *is logarithmic stability estimates. This logarithmic order is also given in* [2, 6, 16, 20, 21].

4. Notice that the error is in [19] *(See Theorem 2.6 , page 5).*

$$\|u(0) - u_\epsilon(0)\| \quad \le \quad NTe^{kT}\left(1 + \ln\frac{T}{\epsilon})\right)^{-1}. \tag{2.16}$$

In [2] *(see Theorem 4.14, p.12), the authors established the optimal order error of their method. With the condition $\|Au(0)\|^2 = \int_\gamma^{+\infty} \lambda^2 e^{2T\lambda}\, d\|E_\lambda\varphi\|^2 < \infty$, and that $\gamma \ge 1$, they estimated $\|u_\sigma(0) - u(0)\|^2$ as follows*

$$\|u_\sigma(0) - u(0)\|^2 \le 2\big((\frac{T}{1 + \ln(\frac{\gamma T}{\beta})})^2 + T\alpha\big)\|Au(0)\|^2. \tag{2.17}$$

The error orders are same in (2.14).

5. It follows from (2.15) *we obtain the Holder stability. As we know, the error of Holder form is the optimal error. Thus,the convergence to zero of ϵ^α, $(0 < \alpha < 1)$ is quickly than logarithmic terms. We note again such order is not considered in* [16]. *Comparing* (2.15) *and* (2.16) *with* (2.17) *and the results obtained in* [16, 18, 19], *we realize* (2.15) *is sharp and the best known estimate. The convergence to zero of ϵ^α, $(0 < \alpha < 1)$ is quickly than logarithmic terms. This is generalization of many discussed results.*

3. Proof of the main results.

Proof of Lemma 2.1.
If the problem (1.1) has a unique solution u then

$$u(t) = e^{-tA}u(0). \tag{3.1}$$

Then $u(T) = f = e^{-tA}u(0)$. It follows that

$$\|u(.,0)\|^2 = \int_0^\infty e^{2T\lambda} d\|E_\lambda f\|^2 < \infty.$$

If we get (7), then define v be as the function

$$v = \int_0^\infty e^{T\lambda} dE_\lambda f.$$

Consider the problem

$$\begin{cases} u_t + Au = 0, \\ u(0) = v, \end{cases} \tag{3.2}$$

It is clear to see that (3.2) is the direct problem so it has a unique solution u. We have

$$u(t) = \int_0^\infty e^{-t\lambda} e^{T\lambda} dE_\lambda f. \tag{3.3}$$

Let $t = T$ in (3.3), we have

$$u(T) \quad = \quad \int_0^\infty e^{-T\lambda} e^{T\lambda} dE_\lambda f = f.$$

Hence, u is the unique solution of (1.1).

Proof of Theorem 2.2.
It is well known that for all $t \in [0, T]$,

$$v_\epsilon(t) - w_\epsilon(t) \quad = \quad \int_0^\beta e^{\lambda(T-t)} dE_\lambda (f_\epsilon - g_\epsilon). \tag{3.4}$$

Using (3.4), we obtain

$$\begin{aligned} \|v_\epsilon(t) - w_\epsilon(t)\|^2 \quad &\leq \quad e^{2(T-t)\beta} \int_0^\infty d\|E_\lambda (f_\epsilon - g_\epsilon)\|^2 \\ &\leq \quad e^{2(T-t)\beta} \|f_\epsilon - g_\epsilon\|^2. \end{aligned}$$

This inequality follows the solution of the problem (2.4) depend continuously on φ and Theorem 2.2 is proved.

Proof of Theorem 2.3.
The functions $u(t), u_\epsilon(t)$ have the expansion

$$u(t) \quad = \quad \int_0^\infty e^{\lambda(T-t)} dE_\lambda f. \tag{3.5}$$

$$u_\epsilon(t) \quad = \quad \int_0^\infty e^{\lambda(T-t)} \chi_{[0,\beta]} dE_\lambda f. \tag{3.6}$$

Hence, we get

$$u(t) - u_\epsilon(t) \quad = \quad \int_\beta^\infty e^{\lambda(T-t)} dE_\lambda f = \int_0^\infty e^{-\lambda t} \chi_{[\beta,\infty]} e^{\lambda T} dE_\lambda f.$$

Then

$$\|u(t) - u_\epsilon(t)\|^2 \quad \leq \quad \int_0^\infty \left(e^{-\lambda t} \chi_{[A_\epsilon,\infty]}\right)^2 \left(e^{\lambda T} dE_\lambda f\right)^2.$$

Using

$$\left(e^{-\lambda t} \chi_{[\beta,\infty]}\right)^2 \leq e^{-2t\beta}$$

and

$$\|u(0)\|^2 = \int_0^\infty \left(e^{\lambda T} dE_\lambda f\right)^2.$$

we obtain

$$\|u(t)-u_\epsilon(t)\|^2 \quad \le \quad e^{-2t\beta}\|u(0)\|^2.$$

Using (2.5) and the triangle inequality, we get

$$\begin{aligned}\|u(t)-v_\epsilon(t)\| &\le \|u(t)-u_\epsilon(t)\|+\|u_\epsilon(t)-v_\epsilon(t)\|\\ &\le e^{-t\beta}\|u(0)\|+e^{(T-t)\beta}\epsilon.\end{aligned}$$

This completes the proof of Theorem 2.3.

Proof of Theorem 2.4

Proof a. Since (3.5) and (3.6), we have

$$\begin{aligned}u(t)-u_\epsilon(t) &= \int_0^\infty e^{\lambda(T-t)}\chi_{[\beta,\infty]}dE_\lambda f\\ &= \int_0^\infty \lambda^{-\beta}e^{\lambda(T-t)}\lambda^\beta\chi_{[\beta,\infty]}dE_\lambda f.\end{aligned}$$

Then

$$\begin{aligned}\|u(t)-u_\epsilon(t)\|^2 &= \int_0^\infty \left(\lambda^{-\beta}\chi_{[\beta,\infty]}\right)^2\left(e^{\lambda(T-t)}\lambda^\beta dE_\lambda f\right)^2\\ &\le \beta^{-2p}\int_0^\infty \lambda^{2p}d\|E_\lambda u(t)\|^2.\end{aligned}$$

Using (2.5) and the triangle inequality, we get

$$\begin{aligned}\|u(t)-v_\epsilon(t)\|\le\|u(t)-u_\epsilon(t)\|+\|u_\epsilon(t)-v_\epsilon(t)\| &\le I_1\beta^{-p}+e^{(T-t)\beta}\epsilon\\ &\le \epsilon^{\frac{at}{T}+1-a}+(\frac{T}{a})^pI_1\left(\ln(\frac{1}{\epsilon})\right)^{-p}.\end{aligned}$$

Proof b. Since (3.5) and (3.6), we also have

$$\begin{aligned}u(t)-u_\epsilon(t) &= \int_0^\infty e^{\lambda(T-t)}\chi_{[\beta,\infty]}dE_\lambda f\\ &= \int_0^\infty e^{-q\lambda}e^{\lambda(T-t)}e^{q\lambda}\chi_{[\beta,\infty]}dE_\lambda f.\end{aligned}$$

Then

$$\begin{aligned}\|u(t)-u_\epsilon(t)\|^2 &= \int_0^\infty \left(e^{-q\lambda}\chi_{[\beta,\infty]}\right)^2\left(e^{\lambda(T-t)}e^{q\lambda}dE_\lambda f\right)^2\\ &\le e^{-2q\beta}\int_0^\infty e^{2q\lambda}d\|E_\lambda u(t)\|^2.\end{aligned}$$

Using (2.5) and the triangle inequality, we get

$$\begin{aligned}\|u(t)-v_\epsilon(t)\|\le\|u(t)-u_\epsilon(t)\|+\|u_\epsilon(t)-v_\epsilon(t)\| &\le I_2e^{-2q\beta}\int_0^\infty e^{2q\lambda}d\|E_\lambda u(t)\|^2+e^{(T-t)\beta}\epsilon\\ &\le \epsilon^{\frac{q}{T+q}}\left(\epsilon^{\frac{t}{T+q}}+I_2\right).\end{aligned}$$

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