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APPROXIMATING SOLUTIONS FOR THE SYSTEM OF ϕ-STRONGLY ACCRETIVE OPERATOR EQUATIONS IN REFLEXIVE BANACH SPACE

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Abstract. The purpose of this paper is to study a strong convergence of multi-step iterative scheme to a common solution for a finite family of ϕ-strongly accretive operator equations in a reflexive Banach space with weakly continuous duality map. As a consequence, the strong convergence theorem for the multi-step iterative sequence to a common fixed point for finite family of ϕ-strongly pseudocontractive mappings are also obtained. The results presented in this paper thus improve and extend the corresponding results of Inchan [7, 8], Kang [10] and many others.

1. Introduction

Mann [15] and Ishikawa [9] iteration processes have been studied extensively by various authors for approximating the solutions of nonlinear operator equations in Banach spaces (e.g. [18] and the references therein). Liu [12], Osilike [19] and Xu [20] introduced the concepts of Ishikawa and Mann iterative processes with errors for nonlinear strongly accretive mappings in uniformly smooth Banach spaces. It is well known that any strongly accretive and strongly pseudocontractive operators are ϕ-strongly accretive and ϕ-strongly pseudocontractive respectively, but the converse do not hold [18]. Also, every ϕ-strongly pseudocontractive map with a nonempty fixed point set is ϕ-hemicontractive. In [4], Chidume and Osilike constructed an operator which is ϕ-hemicontractive but not ϕ-strongly pseudocontractive. Many authors extended the results for a more general class of ϕ-strongly accretive operator(e.g. [11, 13, 18, 21] and the references therein).
Recently, Kang [10] studied the iterative approximation of solution of a demicontinuous ϕ-strongly accretive operator in a uniformly smooth Banach space, improving many of the previous results et.al. [18, 20].

On the other hand, Noor [17] suggested and analyzed three-step iteration process introduced by Noor [16], for solving the nonlinear strongly accretive operator equation in a uniformly smooth Banach space. It has been shown in [6] that the three-step iterative scheme gives better numerical results than the two-step and

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one-step.
Motivated by above facts, Inchan [7] introduced and analyzed a multi-step iterative scheme with errors for approximating common solution of nonlinear strongly accretive operator equation.

Let K be nonempty convex subset of a uniformly smooth Banach space E and let $T_1, T_2, \cdots, T_N : K \to K$ be mappings. For any given $x \in K$, and a fixed positive integer N, the sequences $\{x_n\}$ defined by

$$\begin{aligned} x_1 &\in K, \\ x_n^1 &= a_n^1 x_n + b_n^1 T_1 x_n + c_n^1 u_n^1, \\ x_n^2 &= a_n^2 x_n + b_n^2 T_2 x_n^1 + c_n^2 u_n^2 \\ &\vdots \\ x_{n+1} = x_n^N &= a_n^N x_n + b_n^N T_N x_n^{N-1} + c_n^N u_n^N, \quad n \geq 1, \end{aligned} \tag{1.1}$$

where $\{a_n^1\}, \cdots, \{a_n^N\}, \{b_n^1\}, \cdots, \{b_n^N\}, \{c_n^1\}, \cdots, \{c_n^N\}$ are sequences in $[0,1]$ with $a_n^i + b_n^i + c_n^i = 1$ for all $i = 1, 2, \cdots, N$ and $\{u_n^1\}, \cdots, \{u_n^N\}$ are bounded sequence in K.
This iteration scheme (1.1) is called the multi-step iteration with errors [7]. These iterations introduce the Mann, Ishikawa, Three step iterations as a special case.
If N=3, $T_1 = T_2 = T_3 = T, a_n = a_n^3, b_n = b_n^2, c_n = c_n^1$ and $c_n^i = 0 \; \forall \; i = 1, 2, \cdots, N$ then (1.1) reduces to the three-step iterations defined by Noor [17]:

$$\begin{aligned} x_{n+1} &= (1 - a_n)x_n + a_n T y_n, \\ y_n &= x_n^2 = (1 - b_n)x_n + b_n T z_n \\ z_n &= (1 - c_n)x_n + c_n T x_n, \quad n \geq 1, \end{aligned} \tag{1.2}$$

where $\{a_n\}, \{b_n\}, \{c_n\}$ are real sequences in $[0,1]$.
If N=2, $T_1 = T_2 = T, a_n^{'} = a_n^1, b_n^{'} = b_n^1, c_n^{'} = c_n^1, a_n = a_n^2, b_n = b_n^2$ and $c_n = c_n^2$ then (1.1) reduces to the Ishikawa iteration process with errors defined by Xu [20]:

$$\begin{aligned} x_{n+1} &= a_n x_n + b_n T y_n + c_n u_n, \\ y_n &= a_n^{'} x_n + b_n^{'} T x_n + c_n^{'} v_n, \quad n \geq 1, \end{aligned} \tag{1.3}$$

where $\{a_n\}, \{a_n^{'}\}, \{b_n\}, \{b_n^{'}\}, \{c_n\}, \{c_n^{'}\}$ are real sequences in $[0,1]$ satisfying the conditions $a_n + b_n + c_n = 1 = a_n^{'} + b_n^{'} + c_n^{'}$ for all $n \geq 1$ and $\{u_n\}, \{v_n\}$ are bounded sequences in K.

It is our purpose in this paper to establish strong convergence theorem of multi-step [7] for approximating common solution of nonlinear ϕ-strongly accretive operator equations and corresponding common fixed points of nonlinear ϕ-strongly pseudocontractive mappings in a reflexive Banach space with weakly continuous duality mapping, thus extending and improving the corresponding results of Inchan [7, 8], Kang [10] and many others to a finite family and in a more general space.

2. Preliminaries

Let E be a real Banach space with dual E^*. The *normalized duality mapping* from E to 2^{E^*} is defined by

$$J(x) := \{x^* \in E^* : \langle x, x^* \rangle = \|x\|^2, \|x\| = \|x^*\|\},$$

where $\langle ., . \rangle$ denotes the duality pairing between the elements of E and E^*.

Definition 2.1. ([2]) A mapping $A\colon D(A) = E \to E$ is said to be accretive if for all $x, y \in E$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Ax - Ay, j(x-y) \rangle \geq 0.$$

The mapping A is said to be strongly accretive if there exists a constant $k \in (0,1)$ such that for all $x, y \in E$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Ax - Ay, j(x-y) \rangle \geq k\|x-y\|^2$$

and is said to be ϕ- strongly accretive [18] if there is a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that for any $x, y \in E$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Ax - Ay, j(x-y) \rangle \geq \phi(\|x-y\|) \|x-y\|.$$

Definition 2.2. ([3]) The mapping $T : E \to E$ is called pseudocontractive if for all $x, y \in E$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \leq \|x-y\|^2.$$

The mapping T is pseudocontractive if and only if $(I-T)$ is accretive and is strongly pseudocontractive (respectively, ϕ-strongly pseudocontractive) if and only if $(I-T)$ is strongly accretive (respectively, ϕ-strongly accretive).

Definition 2.3. The mapping $T : E \to E$ is called ϕ-hemicontractive if $F(T) \neq \phi$ and there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that for any $x \in E, q \in F(T)$, there exists $j(x-q) \in J(x-q)$ such that

$$\langle Tx - q, j(x-q) \rangle \geq \|x-y\|^2 - \phi(\|x-y\|) \|x-y\|.$$

Definition 2.4. Recall that a gauge is a continuous strictly increasing function $\varphi : [0,\infty) \to [0,\infty)$ such that $\varphi(0) = 0$ and $\lim_{r\to\infty} \varphi(r) = \infty$. Associated with a gauge φ is the duality map [2] $J_\varphi : X \to X^*$ defined by

$$J_\varphi(x) = \{x^* \in X^* : \langle x, x^* \rangle = \|x\|\varphi(\|x\|), \|x^*\| = \varphi(\|x\|)\}, x \in X.$$

Clearly the normalized duality map J corresponds to the gauge $\varphi(t) = t$. Browder [2] initiated the study of certain classes of nonlinear operators by means of a duality map J_φ. It also says that a Banach space X has a weakly continuous duality map if there exists a gauge φ for which the duality map J_φ is single valued and weak-to-weak* sequentially continuous (i.e. if $\{x_n\}$ is a sequence in X weakly convergent to a point x, then the sequence $\{J_\varphi(x_n)\}$ converges weak*ly to $J_{\varphi(x)}$). Set for $t \geq 0$,

$$\Phi(t) = \int_0^t \varphi(r)dr.$$

Then it is known that Φ is a convex function and

$$J_\varphi(x) = \partial\Phi(\|x\|), x \in X,$$

where ∂ denotes the sub-differential in the sense of convex analysis.

We shall need the following results.

Lemma 2.5. ([14]) *Suppose that E is an arbitrary Banach space and $A : E \to E$ is a continuous ϕ-strongly accretive operator. Then the equation $Ax = f$ has a unique solution for any $f \in E$.*

Lemma 2.6. ([5]) *Assume that X has a weakly continuous duality map J_φ with gauge φ. Then for all $x, y, \in X$, there holds the inequality*

$$\Phi(\|x + y\|) \leq \Phi(\|x\|) + \langle y, J_\varphi(x + y)\rangle.$$

Lemma 2.7. ([12]) *Let $\{\alpha_n\}_{n=0}^\infty, \{\beta_n\}_{n=0}^\infty$ and $\{\gamma_n\}_{n=0}^\infty$ be three nonnegative real sequences satisfying the inequality*

$$\alpha_{n+1} \leq (1 - w_n)\alpha_n + \beta_n + \gamma_n,\ \forall\ n \geq 0,$$

where $\{w_n\}_{n=0}^\infty \subset [0, 1], \sum_{n=0}^\infty w_n = +\infty$ and $\sum_{n=0}^\infty \gamma_n < +\infty$. Then $\lim_{n\to\infty} \alpha_n = 0$.

3. Main Results

Theorem 3.1. *Let E be a reflexive Banach space with weakly continuous duality map J_φ with gauge φ and let $\{A_i\}_{i=1}^N : E \to E$ be continuous ϕ-strongly accretive operators. Let for $i = 1, \cdots, N$, $\{u_n^i\}_{n=1}^\infty$ be bounded sequences in E and $\{a_n^i\}_{n=1}^\infty, \{b_n^i\}_{n=1}^\infty, \{c_n^i\}_{n=1}^\infty$ be real sequences in $[0, 1]$ satisfying $(i) a_n^i + b_n^i + c_n^i = 1, (ii) \sum_{n=1}^\infty b_n^1 = +\infty, (iii) \sum_{n=1}^\infty c_n^i < \infty, (iv) \lim_{n\to\infty} b_n^i = 0,\ \forall\ i = 1, \cdots, N$ and $n \geq 1$. For any given $f, x_1 \in E$, define $\{S_i\}_{i=1}^N : E \to E$ by $S_i x = x - A_i x + f,\ \forall\ i = 1, \cdots, N$, and the iterative sequence $\{x_n\}_{n=1}^\infty$ with errors be defined by*

$$\begin{aligned}
&x_1 \in E,\\
&x_n^1 = a_n^1 x_n + b_n^1 S_1 x_n + c_n^1 u_n^1,\\
&x_n^2 = a_n^2 x_n + b_n^2 S_2 x_n^1 + c_n^2 u_n^2\\
&\ \vdots\\
x_{n+1} = &x_n^N = a_n^N x_n + b_n^N S_N x_n^{N-1} + c_n^N u_n^N,\ \ n \geq 1,
\end{aligned} \tag{3.1}$$

If atleast one of the following condition:

$$\textit{each of the sequences}\{x_n^i - A_i x_n^i\}_{n=1}^\infty \textit{are bounded} \tag{3.2}$$

$$\textit{or the sequences}\{A_i x_n^i\}_{n=1}^\infty \textit{are bounded}, \forall\ i = 1, \cdots, N, \tag{3.3}$$

is fulfilled, then the sequence $\{x_n\}_{n=1}^\infty$ converges strongly to the unique solution of the operator equations $\{A_i x\}_{i=1}^N = f$.

Proof. By Lemma 2.5, equation $A_i x = f$ has unique solution $q \in E$ $(i = 1, \cdots, N)$. Each S_i is demicontinuous and q is unique fixed point of S_i $(i = 1, \cdots, N)$.

For any $x, y \in E$, $\exists\ j_\varphi(x - y) \in J_\varphi(x - y)$ such that

$$\langle S_i x - S_i y, j_\varphi(x - y)\rangle \leq \|x - y\|^2(1 - P_i(x, y))$$

where $P_i(x,y) = \frac{\phi_i(\|x-y\|)}{1+\|x-y\|+\phi_i(\|x-y\|)} \in [0,1]$ $i = 1, \cdots, N$.
Let $q \in \bigcap_{i=1}^{N} F(S_i)$, where $F(S_i)$ is the fixed point set of S_i and let $P(x,y) = \inf_{n\geq 0} \min_i \{P_i(x_n, y)\} \in [0,1]$.
Since each $A_i (i = 1, \cdots, N)$ is ϕ-strongly accretive, so that

$$\langle A_i x - A_i y, j_\varphi(x-y)\rangle \geq \|x-y\|\phi_i(\|x-y\|),$$

which implies,

$$\phi_i(\|x-y\|) \leq \|A_i x - A_i y\|$$

also,

$$\begin{aligned}\|S_i x - S_i y\| &\leq \|x-y\| + \|A_i x - A_i y\| \\ &\leq \phi_i^{-1}(\|A_i x - A_i y\|) + \|A_i x - A_i y\|\end{aligned}$$

and

$$\|S_i x - S_i y\| \leq \|x - A_i x\| + \|y - A_i y\|$$

Thus either of (3.2), (3.3) implies $\{S_i x_n^i\}_{n=1}^{\infty}$ are bounded.
For each $i \in \{1, 2, \cdots, N\}$, we have

$$\begin{aligned}\|x_n - x_n^i\| &\leq a_n^i \|x_n - x_n\| + b_n^i \|x_n - S_i x_n^{i-1}\| + c_n^i \|x_n - u_n^i\| \\ &= b_n^i \|x_n - S_i x_n^{i-1}\| + c_n^i \|x_n - u_n^i\| \\ &\to 0 \text{ as } n \to \infty.\end{aligned}$$

Put $d_n^i = b_n^i + c_n^i$, $(i = 1, \cdots, N)$ and

$$D = \max\{\max_{1\leq i\leq N} \sup_{n\geq 1}\{\|u_n^i - q\|\}, \max_{1\leq i\leq N} \sup_{x\in E}\{\|S_i x - q\|\}, \|x_1 - q\|\} \tag{3.4}$$

Clearly $D < \infty$.
Next we will prove that $\forall\, n \in \mathbb{N}, \|x_n - q\| \leq D$.
Infact, it is obviously true for $n = 1$. Using mathematical induction assume the inequality is true for $n = k$. Then for $n = k+1$,

$$\begin{aligned}\|x_{k+1} - q\| &= \|a_k^N x_k + b_k^N S_N x_k^{N-1} + c_k^N u_k^N - q\| \\ &\leq a_k^N \|x_k - q\| + b_k^N \|S_N x_k^{N-1} - q\| + c_k^N \|u_k^N - q\| \\ &\leq (a_k^N + b_k^N + c_k^N) D = D.\end{aligned}$$

So we conclude that

$$\|x_n - q\| \leq D, \ \forall n \geq 1. \tag{3.5}$$

For any $i = 1, 2, \cdots, N$, we see that

$$\begin{aligned}\|x_n^i - q\| &= \|a_n^i x_n + b_n^i S_i x_n^{i-1} + c_n^i u_n^i - q\| \\ &\leq a_n^i \|x_n - q\| + b_n^i \|S_i x_n^{i-1} - q\| + c_n^i \|u_n^i - q\| \\ &\leq (a_n^i + b_n^i + c_n^i) D \\ &= D,\end{aligned}$$

it follows that $\{x_n^i - q\}$ are bounded sequences, for all $i = 1, 2, \cdots, N$.
Consider for $n \geq 1$, using Lemma 2.6

$$\begin{aligned}
\Phi(\|x_n^1 - q\|) &= \Phi(\|a_n^1 x_n + b_n^1 S_1 x_n + c_n^1 u_n^1 - q\|) \\
&\leq \Phi(\|a_n^1(x_n - q)\|) + \langle b_n^1(S_1 x_n - q) + c_n^1(u_n^1 - q), j_\varphi(x_n^1 - q)\rangle \\
&\leq \Phi(\|a_n^1(x_n - q)\|) + b_n^1\langle(S_1 x_n - q), j_\varphi(x_n - q)\rangle \\
&\quad + b_n^1\langle(S_1 x_n - q), j_\varphi(x_n^1 - x_n)\rangle + c_n^1\langle(u_n^1 - q), j_\varphi(x_n^1 - q)\rangle \\
&\leq \Phi(\|a_n^1(x_n - q)\|) + b_n^1(1 - P_1(x_n, q))\|x_n - q\|^2 \\
&\quad + b_n^1\|S_1 x_n - q\|\|j_\varphi(x_n^1 - q)\| + c_n^1\|u_n^1 - q\|\|j_\varphi(x_n^1 - q)\| \\
&\leq a_n^1 \Phi(\|x_n - q\|) + \beta_n^1 + \gamma_n,
\end{aligned}$$

where $\beta_n^1 = b_n^1(1 - P_1(x_n, q))\|x_n - q\|^2 + b_n^1\|S_1 x_n - q\|\|j_\varphi(x_n^1 - q)\|$ and $\gamma_n = c_n^1\|u_n^1 - q\|\|j_\varphi(x_n^1 - q)\|$.
From boundedness of $\{x_n^i - p\}$, it follows by conditions (ii) and (iii) that $\lim_{n\to\infty} \beta_n^1 = 0$ and $\sum_{n=1}^\infty \gamma_n < \infty$.
Next,

$$\begin{aligned}
\Phi(\|x_n^2 - q\|) &= \Phi(\|x_n^2 - x_n^1 + x_n^1 - q\|) \\
&\leq \Phi(\|x_n^1 - q\|) + \langle x_n^2 - x_n^1, j_\varphi(x_n^2 - q)\rangle \\
&\leq \Phi(\|x_n^1 - q\|) + \|x_n^2 - x_n^1\|\|j_\varphi(x_n^2 - q)\| \\
&\leq a_n^1 \Phi(\|x_n - q\|) + \beta_n^2 + \gamma_n
\end{aligned}$$

where $\beta_n^2 = \beta_n^1 + \|x_n^2 - x_n^1\|\|j_\varphi(x_n^2 - q)\|$.
By the proof above, we have $\lim_{n\to\infty} \|x_n^2 - x_n^1\| = 0$ and so it follows that $\lim_{n\to\infty} b_n^2 = 0$.
Next we note that,

$$\begin{aligned}
\Phi(\|x_n^3 - q\|) &= \Phi(\|x_n^3 - x_n^2 + x_n^2 - q\|) \\
&\leq \Phi(\|x_n^2 - q\|) + \langle x_n^3 - x_n^2, j_\varphi(x_n^3 - q)\rangle \\
&\leq \Phi(\|x_n^2 - q\|) + \|x_n^3 - x_n^2\|\|j_\varphi(x_n^3 - q)\| \\
&\leq a_n^1 \Phi(\|x_n - q\|) + \beta_n^3 + \gamma_n
\end{aligned}$$

where $\beta_n^3 = \beta_n^2 + \|x_n^3 - x_n^2\|\|j_\varphi(x_n^3 - q)\|$.
Since $\lim_{n\to\infty} \|x_n^3 - x_n^2\| = 0$, so it follows that $\lim_{n\to\infty} b_n^3 = 0$.
By continuity of above method, there exists nonnegative real sequences $\{\beta_n^N\}$ with $\lim_{n\to\infty} \beta_n^N = 0$ and $\sum_{n=1}^\infty \gamma_n < \infty$ such that

$$\Phi(\|x_{n+1} - q\|) \leq (1 - b_n^1)\Phi(\|x_n - q\|) + \beta_n^N + \gamma_n.$$

Since $1 - b_n^1 < 1$ and $b_n^1 \to 0$ as $n \to \infty$, there exists $N \in \mathbb{N}$ and a real number r such that

$$1 - b_n^1 < r < 1,\ \forall\, n \geq N.$$

Putting $\beta_n = \frac{b_n^1 \beta_n^1}{1-r}$, we get that $\beta_n = o(b_n^1)$. Then

$$\Phi(\|x_{n+1} - q\|) \leq (1 - b_n^1)\Phi(\|x_n - q\|) + \beta_n + \gamma_n.$$

It follows from Lemma 2.7,

$$\lim_{n\to\infty} \Phi(\|x_n - q\|) = 0$$

$$\Rightarrow \lim_{n\to\infty} \|x_n - q\| \to 0.$$

□

Remark. (i) *If the family $\{A_i\}_{i=1}^N$ of mappings be such that $A_1 = A_2 = \cdots = A_N = A$, where A is ϕ-strongly accretive, then the result of* [10] *and the references therein, holds as a special case of our theorem. Thus our result extents* [10] *to a finite family of operators in a more general reflexive Banach space.*

(ii) *The strongly accretive operators in* [7, 8] *and the references therein, are replaced by the more general ϕ-strongly accretive operators.*

Theorem 3.2. *Let E, $\{u_n^i\}_{n=1}^\infty$, $\{a_n^i\}_{n=1}^\infty, \{b_n^i\}_{n=1}^\infty, \{c_n^i\}_{n=1}^\infty$ be as in Theorem 3.1 and let $\{T_i\}_{i=1}^N : E \to E$ be demicontinuous phi-strongly pseudocontractive operators. Then the iterative sequence $\{x_n\}_{n=1}^\infty$ with errors be defined by*

$$\begin{aligned}
&x_1 \in E,\\
&x_n^1 = a_n^1 x_n + b_n^1 T_1 x_n + c_n^1 u_n^1,\\
&x_n^2 = a_n^2 x_n + b_n^2 T_2 x_n^1 + c_n^2 u_n^2\\
&\ \vdots\\
x_{n+1} = &x_n^N = a_n^N x_n + b_n^N T_N x_n^{N-1} + c_n^N u_n^N, \quad n \geq 1,
\end{aligned} \tag{3.6}$$

converges strongly to the unique common fixed point of $\{A_i x\}_{i=1}^N$, if atleast one of the following condition (3.2) or (3.3) is fulfilled.

Proof. Since we know that a mapping T is ϕ-strongly pseudocontractive if and only if $(I - T)$ is ϕ-strongly accretive. Thus the proof follows from Theorem 3.1, setting $S_i = I - T_i$ and $f = 0$. □

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